Insulation (electric)

UNIVERSITY OF ILLINOIS BULLETIN

ISSUED WEEKLY

Vol. XXXII April 2, 1935 No. 31

[Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Acceptance for mailing at the special rate of postage provided for in section 1103, Act of October 3, 1917, authorized July 31, 1918.]

MECHANICAL-ELECTRICAL STRESS STUDIES OF PORCELAIN INSULATOR BODIES

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE UTILITIES RESEARCH COMMISSION

BY

CULLEN W. PARMELEE

AND

JOHN O. KRAEHENBUEHL

INDUSTRIAL ARTS
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Note p7

BULLETIN No. 273
ENGINEERING EXPERIMENT STATION

PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

PRICE: SEVENTY-FIVE CENTS

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ENGINEERING EXPERIMENT STATION

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BY

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Porcelain, Insulating

Insulating materials - Testing

ENGINEERING EXPERIMENT STATION
PUBLISHED BY THE UNIVERSITY OF ILLINOIS, URBANA

3000—3-35—7355

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MECHANICAL-ELECTRICAL STRESS STUDIES OF PORCELAIN INSULATOR BODIES

I. Introduction

1. *Object and Scope of Investigation.**—The investigation reported in this bulletin pertains to a portion of the problem studied under a research on porcelain insulators conducted by the Engineering Experiment Station of the University of Illinois in coöperation with the Utilities Research Commission of Chicago.

The object of the investigation was to determine the relation and correlation between the electrical and mechanical properties of porcelain bodies furnished by manufacturers of high voltage insulators and of similar bodies produced under laboratory conditions. The investigation did not attempt an absolute quantitative study of the various bodies, but rather a comparative study, determining general tendencies.

Since the problem differs materially from those studied in normal dielectric investigations, it was necessary to develop an electrode which would give results consistent with the present tentative standard for testing porcelain, and to use specimens with sections adaptable to the electrical tests. The mechanical strength data obtained in this investigation are relative, and not directly comparable with the values per unit section obtained with the present accepted standard test sections.

2. *Acknowledgments.*—This investigation is a part of the work of the Engineering Experiment Station of the University of Illinois, of which Melvin L. Enger, Dean of the College of Engineering, is Director, and of the Department of Ceramic Engineering, of which C. W. Parmelee, Professor of Ceramic Engineering, is the head. The data were obtained and the manuscript prepared while Milo S. Ketchum was Dean of the College and Director of the Experiment Station.

Acknowledgment is made to Prof. Ellery B. Paine for his coöperation in permitting the use of the high voltage laboratories of the Electrical Engineering Department for testing of specimens, to Messrs. T. N. McVay and C. L. Thompson for their work in the preparation of laboratory bodies, and to Mr. A. J. Monack for the

*This investigation was carried on from June, 1927 to August, 1931.

supervision of the preparation of specimens and the microscopic investigation of the various porcelain bodies tested.

Acknowledgment is also made to the following manufacturers of porcelain insulators: Jeffery-Dewitt Insulator Company, Lapp Insulator Company, Incorporated, Locke Insulator Corporation, R. Thomas & Sons Company, and Westinghouse Electric & Manufacturing Company, for their coöperation in furnishing test specimens of porcelain bodies produced under normal conditions of manufacture.

This investigation was conducted in the Engineering Experiment Station of the University of Illinois, in coöperation with the Utilities Research Commission, Inc., of Chicago, representing the Public Service Company of Northern Illinois, the Commonwealth Edison Company, the Middlewest Utilities Company, the Midland United Company, and the Chicago Rapid Transit Company. The Chairman of this Commission was W. L. Abbott.

The Advisory Committee appointed by the Utilities Research Commission consisted of the members of the engineering and research staffs of the several utilities represented.

II. Test Specimens

3. *Shape of Specimens.*—The porcelain bodies were molded into test specimens of two types, one for compression tests and the other for tension tests. The compression specimens were bars ½ in. x 1½ in. in section and 3 in. in length, with the ends ground parallel after the specimens were burned. The compression heads were placed against these ground ends, and the load was applied in the direction of the long axis. The tension specimens used were a modified form of the type specified for molded material by the A.S.T.M. (C 77-32). Figure 1 shows the dimensions of the tension specimen. All test data presented are for the special specimens used. The mechanical strength was lower than that obtained from specimens complying with the present specifications of the A.S.T.M., (D 116-30) but the standard forms would not lend themselves easily to dielectric tests.

4. *Bodies Tested.*—Test specimens were obtained from the five representative manufacturers previously mentioned, and one set (No. 6) was prepared in the laboratories of the University of Illinois. In order to facilitate the keeping of records each body has been given a number, with the compression and tension tests indicated by letters. These symbols will be used throughout the discussion and the following table gives the system of notation:

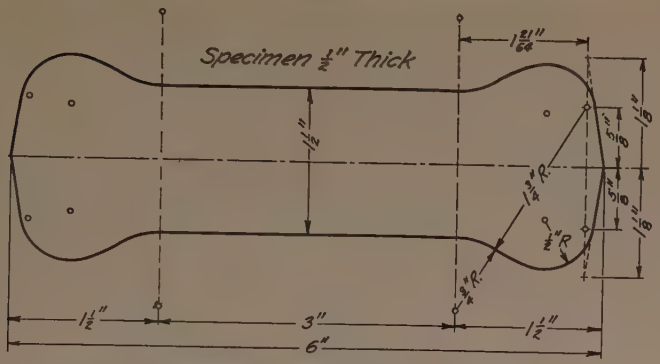


FIG. 1. DIMENSIONS OF TENSION TEST SPECIMEN

Body	Compression Test	Tension Test
1	G	K
2	H	L
3	I	M
4	J	N
5	O	P
6	R	S

The bodies obtained from the manufacturers, with the exception of No. 5, were mixed, molded, and burned at the plant. The manufacturers carried the test specimens through the same process as the actual insulators, and the compression and tension specimens were taken from the same mix in order that any correlation between the tests might be studied. In the case of Body No. 5 the procedure was somewhat changed due to production complications. The body was mixed at the factory and shipped to the University laboratory, where it was molded and partially burned, so that it could be safely shipped back to the factory, and there burned with a regular run of insulators.

5. *Preparation of Laboratory Body (Body No. 6).*—This body had the following composition (proportions by weight):

H. and G, A-1 English China Clay	30 per cent
Dorset English Ball Clay	20 per cent
Ottawa Flint	20 per cent
Buckingham Feldspar	30 per cent

It was mixed and burned at the University laboratory.

The mixture was placed in a blunger with water and reduced to a homogeneous slip in about three hours. The slip was passed through a lawn, 120 mesh to the linear inch, into an agitator which kept the

fluid in motion while it was being fed to a filter press, in which the surplus water was removed, leaving a plastic body.

In order to make the body as homogeneous as possible it was necessary to remove the air bubbles by wedging. After the proper period of wedging the tension and compression specimens were formed by cutting a small slab of clay and forcing it into a brass mold of the proper size and shape. The plastic specimen was removed with a wooden template, shaped to fit the inside of the mold. The specimen was then dried very slowly, otherwise it would have warped and been of little value.

When the specimens were dry they were packed into fireclay saggers and fired in an ordinary kiln to cone 11 to produce a typical electrical porcelain.

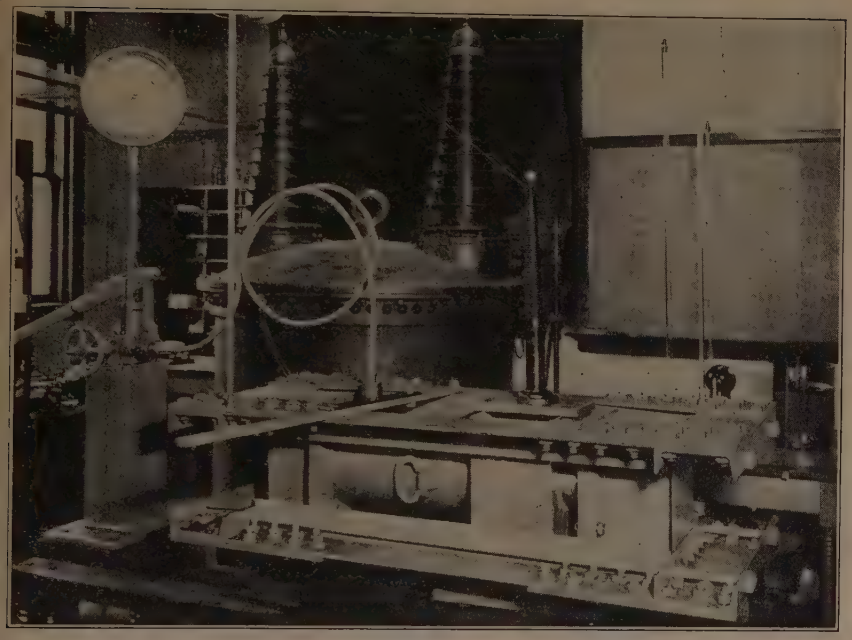
III. Apparatus

6. *Electrical Testing Equipment.*—Test and voltage regulating apparatus used in the investigation were designed by the department and built either in the department shops, in the University shops, or by local firms. The high voltage transformer and auxiliary apparatus was the standard equipment of the Electrical Engineering Department.

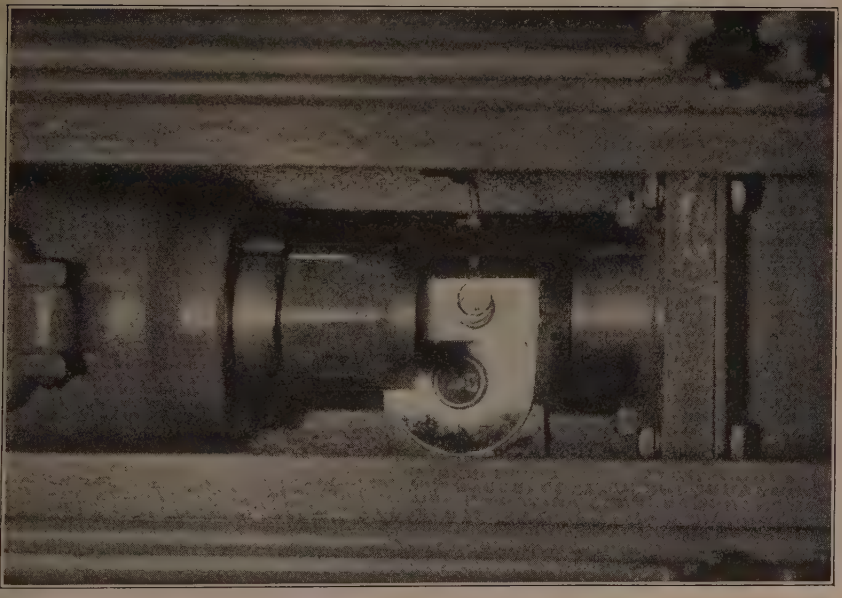
The high voltage equipment consisted of a 200 kv. 100 kv-a. test transformer operated from a d-c.—a-c. motor-generator set. In order to apply the high voltage at a uniform rate of 1 kv. per second it was necessary to design a controlled resistance that would so operate on the field excitation of the generator as to cause the terminal voltage of the transformer to be linear. This equipment, having a set of limit switches, made the operation of applying the voltage uniform and automatic. All specimens were tested under the same operating conditions.

7. *Temperature Control.*—In order that a change in room temperature should not affect the results, all transformer oil used in the test and the specimens themselves were kept at a constant temperature, 35 deg. C., throughout the investigation. The thermostats were of the bi-metal type wound in the form of a spiral. The specimens were preheated in a control oven, and then placed in the testing machine, where they were flooded with transformer oil. The temperature at no time varied more than $\pm$ 2 deg. C.

To insure a uniform temperature of 35 ± 2 deg C. the oil in which the specimen was submerged during a test was withdrawn from the container attached to the testing machine into a movable storage tank. The flow was controlled by gravity. When the storage tank was

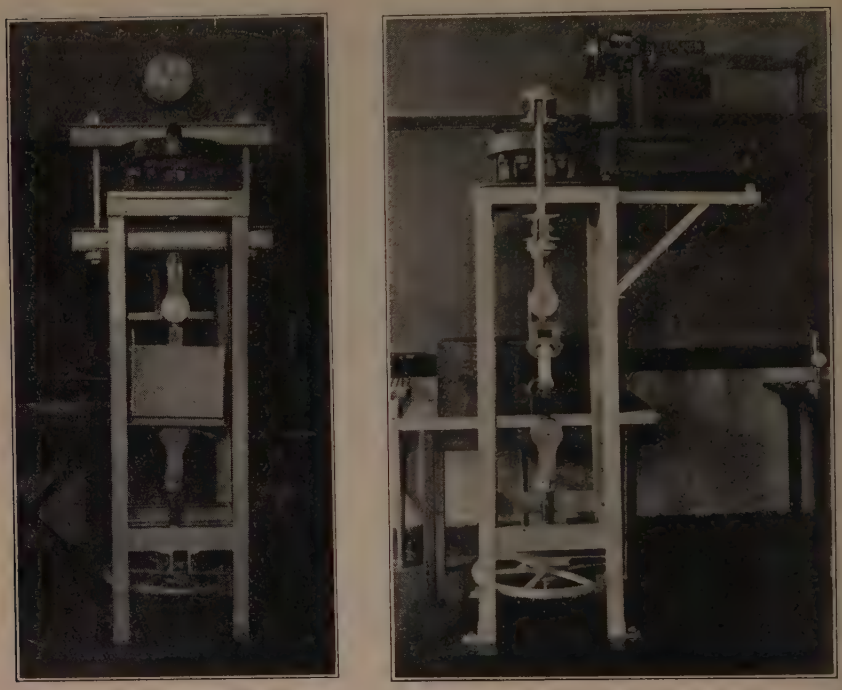


(a) Oil reservoir in place



(b) Oil reservoir removed

FIG. 2. COMPRESSION TESTING MACHINE



(a) Oil container in place (b) Oil container removed

FIG. 3. TENSION TESTING MACHINE

lowered the oil flowed from the testing machine and when it was raised the oil flowed to the testing machine.

The storage tank was equipped with a thermostat and heater to permit of the oil being kept at a constant temperature. The use of the storage tank made possible the placing of the specimen between the heads without the necessity of working in the test oil. In this way the oil was not contaminated by the hands and error in the adjustment of the sample was eliminated.

8. *Testing Machines.*—The compression testing machine was constructed of structural steel sections and a hydraulic jack was used for applying the load. The specimens were placed between a fixed and a movable head. In order that the two heads of the machine should remain parallel, regardless of slight differences in the grinding of the specimens, the movable head contained a ball-and-socket joint, making the head self-aligning. The ends of the specimens were padded with

a thickness of blotting paper to distribute the pressure uniformly. The load was indicated by reading of a pressure gage. To permit of the pressure being kept constant when voltage was applied, a small rope was attached to the pump handle and carried to the control platform for the manipulation of the pump. The pump was grounded to insure against excessive voltages being applied to the rope. Figure 2 shows the details of the test machine with the oil reservoir in place, and with it removed. The high voltage was applied through two hard rubber bushings attached to the reservoir. This arrangement of electrodes permitted the use of 150 kilovolts without flash-over.

The tension machine was also constructed of structural steel; but, in contrast to the compression machine, which was bolted together, the frame of this machine was welded, making a more compact and lighter construction. The load was applied by means of a screw and a hand wheel and measured with an Olsen weighing platform. For the heavier loads it was necessary to increase the leverage of the hand-wheel by the addition of an extension arm. The specimens in this case were padded with blotting paper at each point of contact with the grips. As in the case of the compression machine, the specimens were submerged in a reservoir of transformer oil during the test. The storage tank and bushings being interchangeable were used for both the compression and tension machines. Figure 3 shows the details of the tension apparatus, with the oil container in place and with it removed.

9. *Records.*—A complete record of each test was made on a recording voltmeter. A sample record sheet is shown in Fig. 4. The primary of the test transformer was equipped with a circuit breaker, and the recording voltmeter was attached to the voltmeter winding on the grounded side of the high tension winding. As the voltage was applied to the test specimen the recording voltmeter needle would swing across the scale until the specimen ruptured; the circuit breaker would then open and the needle return to zero. In this manner the break-down voltage was recorded without depending on the operator for a meter reading.

On this same record sheet space was available for making a complete record. The letter and the specimen number indicated the manufacturer and the type of the test specimen, and the only data necessary to determine the load per square inch and the kilovolts per unit of thickness were the physical dimensions of the specimen and the total mechanical load. It will be noted that the record includes a test of the

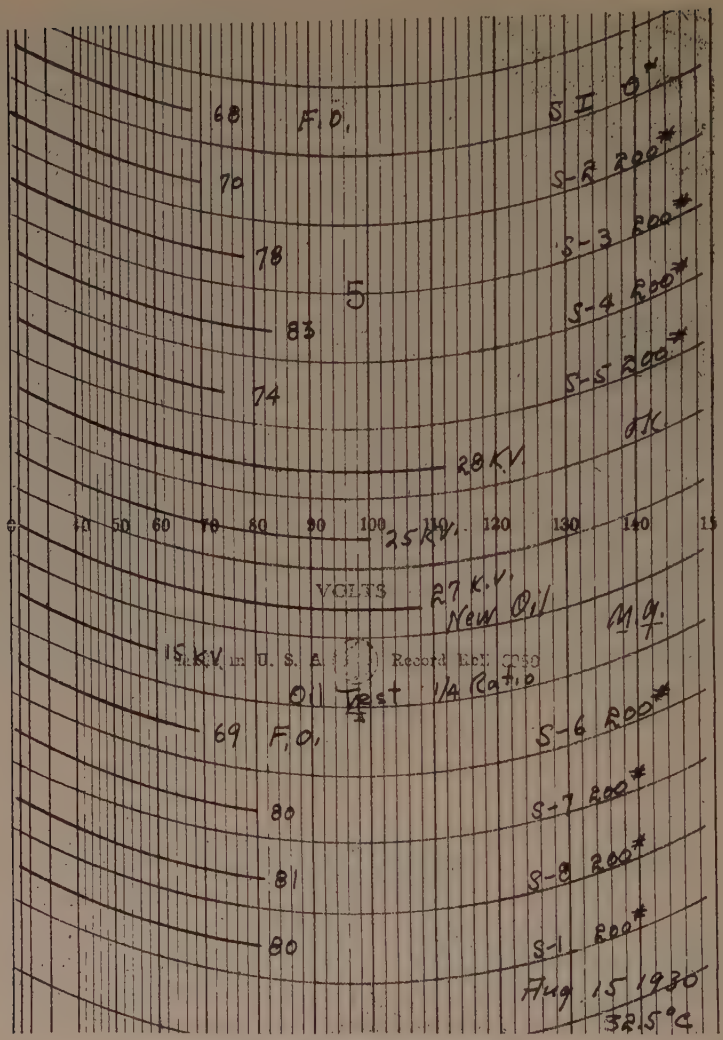


Fig. 4. Sample Record of Test

oil in the container. The oil tests were made after testing the fortieth and the fiftieth specimens, and all oil was discarded after testing the sixtieth specimen. The transformer oil used was Transil Oil 10-C.

Physical measurements of specimens were made with a modified Federal gage which enabled the operator to have both hands free for adjusting specimens and recording readings.

IV. Sealed-In Electrode for Dielectric Tests

10. *Requirements.*—In the testing of porcelain under mechanical stress it is necessary to use an electrode that will permit freedom of movement and that will not flash-over when placed in close proximity to the test heads. The electrode developed for the tests is self-contained, requires no outside equipment, can be made in large quantities, is free to move in any direction with the specimen, and will permit considerable deformation without becoming detached. In characteristics and functioning it does not differ materially from the mercury electrode.

Wood's Metal was substituted for mercury and a copper lead was used for connecting the high voltage terminals. Sealing wax was used for sealing the electrode tube on to the specimen. The specimen was heated so that the sealing wax would not be chilled when coming in contact with the porcelain. Other cements have been tried, but sealing wax is the easiest to handle and can be removed most satisfactorily after the test.

11. *Technique.*—The specimens were cleaned with a stiff fiber brush and washing powder or soap, and then were thoroughly washed in clean water and dried. Before heating the specimens, previous to attaching the tubes, they were dipped in alcohol to remove any trace of foreign material which would keep the sealing wax from making good contact with the specimen.

Since the electrode was cast in a 23-mm. pyrex tube, it was necessary to attach the tube to the specimen. The specimen was heated to 125 or 150 deg C. and the end of the tube was dipped into the melted sealing wax to a depth of one-half to three-quarters of an inch, and then removed. Next, the tube was rotated rapidly in a vertical position so that the molten wax adhering to the end formed a bead. This bead was pressed against the heated porcelain specimen so that the end of the tube was centrally located with respect to the periphery of the specimen. After the tube had been attached to the specimen the assemblage was inverted and a second tube attached exactly opposite to the first. The assemblage was then turned with first one and then the other electrode tube downwards to permit the surplus sealing wax to come in contact with the specimen.

The temperature of the specimen should be such as neither to chill the wax nor to cause it to flow over the surface of the specimen. When the tube is properly attached there will be little sealing wax inside the tube, only a ring at the point of attachment. It does no harm to re-

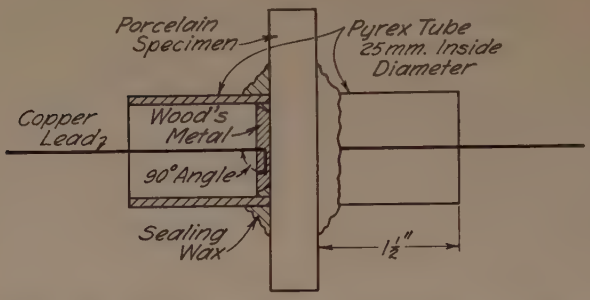


FIG. 5. SPECIMENS WITH SEALED-IN WOOD'S METAL ELECTRODES

move the surplus wax from within the tube with some sharp instrument before it solidifies.

After the tubes had been attached the specimen was ready for the pouring of the electrodes. The specimen was placed in a rack with one of the tubes upwards and a small amount of Wood's Metal (melting point 60 deg. C.) in a molten state poured into the tube. When the metal began to freeze a copper wire, flattened at the end and bent at a 90 deg. angle, was forced into it. When the metal had solidified the wire was held firmly and it was impossible to remove it without considerable force. The specimen was then inverted in the rack, and the other electrode poured. Only a small amount of metal was poured into the electrode tube, for its coefficient of expansion is less than that of glass, and a large mass of metal will crack the glass.

TABLE 1

COMPARATIVE TESTS OF PORCELAIN SPECIMENS WITH DIFFERENT TYPES OF SEALED-IN ELECTRODES

30 specimens in each group

	Type of Electrode		
	A	B	C
Average Dielectric Strength, kv	87.9	86.6	84.2
Median	84.4	85.6	83.5
Range	28.6	22.1	17.4
Mean Deviation	7.4	6.3	3.7
Standard Deviation	9.4	7.5	4.8
Probable Error of the Mean	1.2	1.0	0.6
Per Cent Probable Error of the Mean	1.4	1.2	0.7

A, mercury electrodes A.S.T.M. (D 116-28T)
B, Wood's Metal electrode not heated
C, Wood's Metal electrode—heated to 150 deg. C.

These electrodes can be attached to a large number of specimens at one time; the investment is small, and the method lends itself to production procedure. After assembly, the portion to be tested is sealed against contamination, and the assemblage as a whole, when properly prepared, will withstand rather severe handling.

Figure 5 shows the details of the assemblage, and the relative positions of the various parts. When the electrodes are cast on plate glass they show a surface as smooth as the glass itself, while those removed from the porcelain show the normal irregularities.

The removal of the electrodes for the inspection of the puncture, or for taking measurements, is very simple. The assemblage as a whole is placed in a container filled with gasoline for two or three hours, and at the end of this time the tubes may be readily removed and the Wood's Metal electrode disengaged from the tube. All remaining sealing wax may be removed from the tubes, electrodes, and specimens with clean gasoline. When dry, the specimens are then ready for further investigation. The tubes, after being washed with hot water and washing powder, rinsed in clear hot water, and dried, are available for other tests. The electrodes should likewise be cleaned with hot water and washing powder. After drying, the Wood's Metal may be removed from the copper wire by heat, and, with the exception of the sealing wax, the salvage is complete.

A series of tests was made to compare the Wood's Metal electrodes with the approved mercury sealed-in electrode. Table 1 presents the comparison in a statistical form.

TABLE 2

PERFORMANCE OF WOOD'S METAL SEALED-IN ELECTRODE

A statistical study of 27 groups of porcelain specimens

	Range kv.	Mean Deviation kv.	Standard Deviation kv.	Mean Deviation* kv.
Average Dielectric Strength	23.4	6.6	7.4	6.1
Median	22.0	5.7	6.1	6.0
Range	50.8	13.8	15.8	6.2
Mean Deviation	7.6	2.3	2.5	1.4
Standard Deviation	11.2	3.4	3.8	3.1
Probable Error of the Mean	1.6	0.5	0.5	0.5

*This is for the 23 best samples.

12. *Reliability of Electrodes.*—

Test No. 1

This test was made to determine if there is any error introduced by heating the specimen to the prescribed temperature for sealing and to compare the mercury and the Wood's Metal electrodes. The samples used were approximately one-half inch thick and were made of an electrical porcelain body prepared in the laboratory.

Samples B and C (see Table 1) show lower averages than A, but this is probably due to the fact that the Wood's Metal tends to form sharper points than the mercury, and in sample C there were more sharp edges due to the tapering of the sealing wax as it flowed over the specimen. An inspection of the punctured specimen verifies this opinion, and shows that the breakdown occurred at these sharp points. The one undesirable feature of the sealed-in electrode is that over 85 per cent of the breakdowns occur at the periphery of the electrode. Since the tests are for comparative purposes, this is not as objectionable as it may seem, and the figures shown in the table indicate that either type of electrode gives results within the desired limits for testing porcelain.

It is clearly brought out that heating the specimen is not detrimental to the accuracy of the results but rather tends to make the variation less. As this method was the most satisfactory for the work in hand it was used throughout the investigation.

Test No. 2

To determine what may be expected when using the sealed-in Wood's Metal electrode, an investigation was made on a series of twenty-seven groups consisting of ten samples each. Table 2 gives a

TABLE 3

EFFECTS OF ELECTRODE SHAPE ON SEALED-IN ELECTRODES

30 specimens in each group

	Type of Electrode		
	A	B	C
Average Dielectric Strength, kv	84.3	79.7	75.3
Median	83.9	80.1	75.2
Range	17.3	11.2	14.2
Mean Deviation	4.8	2.3	3.6
Standard Deviation	5.4	3.3	4.2
Probable Error of the Mean	0.7	0.4	0.5

A, Standard Electrode
B, Formed Electrode 3/16 inch radius
C, Sphere Electrode
The electrode B was a turned flat disc with rounded edges, the C electrode was a large ball-bearing. Both B and C were sealed into the tube with sealing-wax.

summary showing the averages of the results of the twenty-seven groups and the variation from the mean.

From Table 2 it will be seen that in a test of ten samples the standard deviation of the average is 7.4 kv. This information should be quite reliable, as the test was performed using samples from four manufacturers and a laboratory body. The approximate thickness of the specimens was one-half inch.

It may be well to note that in only four cases did the mean deviation exceed ten per cent, and the range of the test values in these cases was so large that they would be excluded as unreliable.

Test No. 3

It has been mentioned that 85 per cent of the specimens failed on the periphery of the electrode. This is not a desirable condition, and this last test was conducted to determine, if possible, a shape of electrode that would be more satisfactory. To remove some of the variations caused by a heterogeneous material, such as porcelain, the tests were performed on plate glass one-fourth inch thick. The results of the tests are given in Table 3.

In the case of the electrode B only those samples were accepted for which the puncture was other than at the edge of the electrode. Inspection of the results indicates that the formed electrode is more satisfactory than other types, but that the results obtained with the formed electrode do not justify the labor involved.

13. *Summary.*—To summarize the information gained from these experiments, it may be said that if the electrode is formed with sealing

wax on the heated specimen, and is poured without attempting to obtain any special form, the results as to probable error and the normal distribution of the punctures of the specimens will be satisfactory. In order to guarantee the proper distribution of the results, enough specimens, at any one point of investigation, should be punctured to assure the investigator that the probable error does not exceed three per cent, which is approximately 2 to 2.5 kv. on the half-inch-thick specimens tested. This practice assures a distribution as good as would be obtained from the average investigation of ten specimens taken at random.

The foregoing conclusion follows from the investigation of thirty-nine samples of porcelain containing ten specimens each. On thirty-nine samples of porcelain tested in the laboratory the average of the probable error was 2.8 per cent, with a range of from 2.19 to 3.33 per cent for any similar body. The establishment of the arbitrary value of three per cent appeared to be justified.

All the tests made in the following report were carried out on samples of such a size that the results fell within the probable error limits of three per cent.

V. Mechanical-Dielectric Tests

14. *Statement of Problem.*—The main body of this bulletin deals with the relation between dielectric failure of the specimen and mechanical loading. This may be considered from two viewpoints, (1) the influence of the electrical failure on the strength of the material, and (2) the influence of the loading on the electrical strength.

In order that the series of bodies may be compared on the same basis, it is necessary to establish some definite limit of probable error, for if the limits of error are not controlled the average will have no significance in a comparative plot.

In experimental work, it is frequently assumed that the arithmetic mean indicates the central tendency of any data. Another assumption is that the distribution of the observations made upon any one body is that of a normal frequency curve, and that no factors enter into the observations that would tend to correlate the results. This latter assumption is readily checked after a few observations.

The statistical treatment of the data is not a proof of accuracy, it merely directs interpretation of the results so as to remove biased decisions. The coefficients obtained for samples as small as those dealt with in this experimental work, always less than thirty observations

per point, are not to be accepted as absolute, but only as indicating the presence of a relation and not the degree thereof.

The probable error in each case has been determined by applying "Student's" corrections for small samples. Though other statistical values have been indicated for reference and comparison, only the standard deviation and the probable error have been considered for the analysis of the data, these being used in the determination of the variability of the results and the reliability of the mean.

The effect of the puncture on the strength of the specimens is studied in the first portion of the investigation. In this part of the investigation there is an attempt to answer the question "Does the puncturing of the specimen cause a destruction of the material by forming fissures, or is the temperature increased to such an extent that the material is fused and not materially weakened?" A following section will deal with a microscopic study of the puncture track.

15. *Compression Specimens.*—The initial tests were made on ten specimens of each of the six bodies. Twenty specimens were chosen at random from each shipment received, ten were loaded and punctured, and ten others were loaded without puncturing. Ten specimens is the least number that would give consistent results or means that compared favorably with those obtained from the complete sample, of twenty-seven groups of ten each, investigated for statistical criterion.

Table 4 gives a study of the compression specimens chosen for the investigation, a sufficient number of pieces having been ruptured both mechanically and electrically to insure the desired probable error. The last column in the table gives the rank or the order of the specimens when arranged in order of decreasing strength. Since this type of specimen is not used in standard tests, the loading cannot be considered as representative for the porcelain, but may be used for comparison purposes as the conditions in each instance are the same.

16. *Tension Specimens.*—Table 5 deals with conditions similar to the foregoing except that in this case the specimens are tension specimens. The coefficients of variability indicate that there was not the same deviation from the mean in the case of the tension specimens as was found for the compression specimens. This coefficient was later used in loading the tension specimens very close to the limiting strength, without the loss of a great number by mechanical failure when making dielectric tests.

Table 6 shows the portion of the strength left in the specimen after it has been punctured. The compression specimens retain about 85

TABLE 4

MECHANICAL STRENGTH OF COMPRESSION SPECIMENS BEFORE AND AFTER PUNCTURE

Number of Specimens Tested	Mean lb. per sq. in.	Probable Error of Mean lb. per sq. in.	Standard Deviation lb. per sq. in.	Coefficient of Variability per cent	Body	Median lb. per sq. in.	Mean Deviation lb. per sq. in.	Range	Rank
				Before Puncture					
10	43 600	3100	13 100	30	1	43 500	10 700	38 600	2
10	47 100	4360	18 400	39	2	42 300	16 600	50 600	1
10	38 000	2290	9 670	25	3	36 300	8 840	31 700	4
10	36 800	2780	11 700	32	4	39 400	10 900	27 400	5
10	35 200	1600	6 760	19	5	32 600	5 790	18 600	6
10	38 500	860	3 610	9	6	39 000	3 240	9 900	3
				After Puncture					
10	35 100	3180	13 400	38	1	31 800	12 100	32 500	2
10	41 500	2420	10 200	25	2	36 800	8 800	28 700	1
10	31 100	2420	10 200	33	3	36 000	9 580	25 600	5
10	32 700	1450	6 130	19	4	32 400	5 460	15 300	4
10	30 000	780	3 270	11	5	31 300	2 600	840	6
10	33 200	1780	7 510	23	6	32 100	5 440	23 000	3

This table presents a statistical study of a series of tests to determine the amount of deterioration of the specimen caused by mechanical injury due to the passage of current through the specimen at electrical break-down.

Ten specimens of each body were tested before being punctured and ten were tested after electrical break-down. Then sufficient other specimens were tested, of such bodies as showed unusually high probable error, to reduce the probable error to less than ten per cent.

TABLE 5

MECHANICAL STRENGTH OF TENSION SPECIMENS BEFORE AND AFTER PUNCTURE

Number of Specimens Tested	Mean lb. per sq. in.	Probable Error of the Mean lb. per sq. in.	Standard Deviation lb. per sq. in.	Coefficient of Variability per cent	Body	Median lb. per sq. in.	Mean Deviation lb. per sq. in.	Range	Rank
				Before Puncture					
10	1820	41	173	10	1	1720	147	530	4
10	2300	44	187	8	2	2260	152	500	1
10	2020	71	298	15	3	1200	266	780	2
10	1950	67	283	15	4	1960	282	600	3
10	1370	63	267	19	5	1310	196	920	5
10	1340	39	165	12	6	1340	134	580	6
				After Puncture					
10	1550	65	273	18	1	1690	216	910	2
10	1450	4	15	1	2	1450	15	30	3
10	1300	92	389	30	3	1420	368	870	4
10	1630	1	3	0.2	4	1640	3	70	1
10	1190	2	11	0.9	5	1240	10	250	5
10	1070	43	180	17	6	1080	142	520	6

This table presents a statistical study of a series of tests to determine the amount of deterioration of the specimen caused by mechanical injury due to the passage of current through the specimen at the electrical break-down.

Ten specimens of each body were tested without being punctured and ten were tested after electrical break-down. Then sufficient other specimens were tested, of bodies showing unusually high probable error, to reduce the error to less than five per cent.

TABLE 6

PERCENTAGE OF MECHANICAL STRENGTH REMAINING AFTER PUNCTURE

Body	Percentage of Compressive Strength Remaining	Percentage of Tensile Strength Remaining
1	81	85
2	88	63
3	82	64
4	89	84
5	85	87
6	86	80
Average	85	77

This table gives the percentage of the original mechanical strength of tension and compression specimens remaining after puncture.

The "Rank Correlation" of the percentages given in the table for compression and tension specimens is —0.34 (by "Footrule" formula).

per cent of their strength, while the tension specimens retain only 77 per cent of their strength. A study of the rank correlation between the tension and compression specimens indicates that there is a very slight negative relationship, or that a conclusion drawn for compression would probably be reversed in tension. This fact does not seem to agree with the accepted principle of the breakdown of compression specimens, which, in such a brittle material, should be a tension failure. This disagreement may be due, in part, to the shape of the specimens, as neither type conforms to the best shape for loading tests.

17. *Dielectric Strength.*—In order to compare the dielectric breakdown voltage with the results of the mechanical tests, specimens were tested to determine their dielectric strength at no load. In order that the subsequent results might be consistent as to arrangement and electrostatic field the specimens were tested in the loading machines surrounded by the grips or heads just as they would be located when load was applied. Table 7 gives the results of the tests. In this instance, as in all other determinations of the dielectric strength, the probable error has been held within three per cent or less. The mean is accepted as the most probable central or representative value. An inspection of the table shows differences of dielectric strength in the same body for the compression and tension test pieces. In addition to the fact that the tests deal with a rather unreliable material, there is a second factor which might be responsible for the differences, the electrostatic field distribution. However, the rank is practically the same

TABLE 7

DIELECTRIC STRENGTH OF SPECIMENS AT ZERO MECHANICAL LOAD

Number of Specimens Tested	Mean kv.	Probable Error of Mean kv.	Standard Deviation kv.	Coefficient of Variability per cent	Body	Median kv.	Mean Deviation kv.	Range	Rank
				Compression Specimens					
12	88	2.6	12.1	14	1	86	9.2	43	3
12	97	2.2	10.2	10	2	95	9.4	39	1
11	86	2.5	11.1	13	3	87	9.8	32	4
10	93	1.6	6.9	7	4	92	5.5	26	2
13	75	2.2	10.6	14	5	73	10.2	44	6
10	79	1.7	7.3	9	6	80	5.4	27	5
				Tension Specimens					
25	90	2.7	18.7	21	1	87	15.4	68	3
21	103	3.0	18.9	18	2	95	14.7	56	1
10	83	2.5	10.6	13	3	80	7.9	36	4
18	93	2.8	16.4	18	4	86	13.4	54	2
10	79	2.0	8.5	11	5	78	6.7	28	5
10	72	2.2	9.3	13	6	69	6.8	34	6

Ten specimens of each body were tested. Then a sufficient number of other specimens of bodies showing unusually high probable error were tested, to reduce the probable error to less than three per cent.

TABLE 8

PERCENTAGE OF TENSION SPECIMENS THAT FAILED MECHANICALLY DURING DIELECTRIC TESTS

Body	Percentage that Failed at Load of			
	800 lb. per sq. in.	1070 lb. per sq. in.	1330 lb. per sq. in.	1600 lb. per sq. in.
1	3	5	14	14
2			4	
3	7	5	6	8
4				8
5		3	4	
6	3	8	10	

The number of mechanical failures is cumulative, the failures at a lower load being added to the failures of the next higher load. This percentage, therefore, represents the total failures at all loads up to and including the specific load.

in each instance, there being one reversal of position, and in this case it will be noted that there is a similar reversal in the probable error.

During the test to determine the effect of tension on the dielectric strength of the material, it was noted that some of the specimens failed mechanically at the same time as they were punctured, and it was also noted that this failure occurred at light loads in some specimens, while in others this type of failure did not occur until close to the ultimate load. As the liability to mechanical failure at the period of dielectric breakdown is an interesting property of the body, the observed data are tabulated in Table 8. It was possible to carry only three specimens up to 1600 pounds per square inch. Table 9 indicates that this property is independent of the ultimate mechanical load that may be applied to the specimen. It is unfortunate that a study could not be made of this property in the compression specimens, but up to the present time there has not been devised an electrode or a cement for the sealed-in electrode that will withstand the same percentage of the ultimate load in compression that the sealing wax electrodes were able to withstand in the case of the tension specimens.

18. *Summary.*—Table 9 gives a summary of the ranking of each of the various bodies under the different tests. It affords an easy means of determining the correlation of the properties of the bodies.

In general, it may be said that the bodies hold about the same ranking, from high to low, for all the properties studied. The agreement in dielectric strength for the test pieces of the two different shapes (E and F) shows a coefficient of 0.97, which would be con-

TABLE 9

SUMMARY OF RANK AND CORRELATIONS FOR VARIOUS BODIES TESTED

Body	Rank*						
	A	B	C	D	E	F	G
1	2	4	2	2	3	3	6
2	1	1	1	3	1	1	3
3	4	2	5	4	4	4	4
4	5	3	4	1	2	2	1
5	6	5	6	5	6	5	2
6	3	6	3	6	5	6	5

Correlations† (by "Footrule" formula)

Groups	Coefficient of Correlation	Groups	Coefficient of Correlation
A and C	0.97	A and E	0.71
B and D	0.51	B and F	0.88
E and F	0.97	B and G	−0.05
A and B	0.24	F and G	0.24

*A—Rank of unpunctured compression specimens (last column, Table 4) for compressive strength.

B—Rank of unpunctured tension specimens (last column, Table 5) for tensile strength.

C—Rank of punctured compression specimens (last column, Table 4) for compressive strength.

D—Rank of punctured tension specimens (last column, Table 5) for tensile strength.

E—Rank of compression specimens (last column, Table 7) for dielectric strength at zero mechanical load.

F—Rank of tension specimens (last column, Table 7) for dielectric strength at zero mechanical load.

G—Rank of specimens which failed under mechanical load at time of puncture (Table 8); a loading of 1330 lb. per sq. in. was taken for this table as only one-half of the specimens would withstand a higher mechanical stress.

†The correlations were determined by using the "Rank Method," and the results indicate the *existence* of correlation, and *not the closeness of relationship.*

sidered a perfect agreement; this is as it should be, for the bodies, except for shape, received the same treatment in their manufacture. The comparison of the punctured and the unpunctured compression specimens (A and C) shows an equal degree of relationship, indicating that the electrical damage to the specimen is uniform and that the same degree of weakening might be expected in any specimen. In the case of the punctured and unpunctured tension specimens (B and D) there is a positive relationship, but the indication of a uniform weakening in all the specimens is only just present, without being very marked; and it may be accepted that there is an equal chance that the specimen will not be weakened by some fixed percentage, but is very likely to change its ranking in any series of comparative tests.

How the compressive and tensile strengths of the bodies compare (A and B) is shown by a correlation of only 0.24, indicating a relationship which cannot be considered marked. This may be due

Table 10

Dielectric Strength of Tension Specimens Under Mechanical Stress

Number of Test Specimens	Mean kv.	Probable Error of the Mean kv.	Per cent Probable Error of the Mean	Standard Deviation kv.	Coefficient of Variability per cent	Load lb. per sq. in.	Median kv.	Mean Deviation kv.	Range kv.
				Body No. 1 (K)					
26	90	2.6	2.9	18.7	21	0	87	15.4	68
15	93	2.7	3.0	15.1	16	270	88	11.5	52
10	84	1.1	1.3	4.7	6	540	84	3.8	17
10	88	2.2	2.5	9.3	11	800	84	8.2	29
10	87	1.8	2.0	7.4	9	1070	86	6.4	22
18	95	2.9	3.0	16.9	18	1330	88	13.7	55
10	104	3.0	2.9	13.4	13	1600	102	11.8	41
				Body No. 2 (L)					
21	103	3.0	2.9	18.9	18	0	95	14.7	56
10	100	2 5	2.5	10.8	11	270	99	9.2	31
10	101	2 3	2.3	9.6	10	540	101	7.8	32
10	98	2.8	2.9	12.0	12	800	97	10.3	35
10	103	2.6	2.5	10.8	11	1070	100	9.3	30
11	104	3.0	2.9	13.3	13	1330	104	10.4	52
				Body No. 3 (M)					
10	83	2.5	3.0	10.6	13	0	80	7.9	36
10	84	1.5	1.7	6.1	7	270	83	4.5	23
10	87	1.0	1.1	4.2	5	540	87	3.0	17
10	87	1.7	1.9	7.0	8	800	86	5.0	23
10	97	2.3	2.3	9.6	10	1070	93	8.0	32
12	92	2.7	2.9	12.7	14	1330	90	9.3	46
10	86	2.2	2.5	9.2	11	1600	83	7.8	28

TABLE 10 (Concluded)

DIELECTRIC STRENGTH OF TENSION SPECIMENS UNDER MECHANICAL STRESS

Number of Test Specimens	Mean kv.	Probable Error of the Mean kv.	Per cent Probable Error of the Mean	Standard Deviation kv.	Coefficient of Variability per cent	Load lb. per sq. in.	Median kv.	Mean Deviation kv.	Range kv.
Body No. 4 (N)									
18	93	2.8	3.0	16.4	18	0	86	12.8	54
16	98	2.8	2.9	16.1	17	270	95	12.1	55
10	97	1.9	2.0	8.0	8	540	96	6.6	29
10	97	2.7	2.8	11.6	12	800	99	9.1	41
15	92	2.6	2.8	14.9	16	1070	90	11.4	54
10	93	2.3	2.4	9.6	10	1330	94	8.2	27
14	92	2.7	2.9	14.2	15	1600	85	12.3	41
Body No. 5 (P)									
10	79	1.6	2.0	6.7	11	0	78	8.5	28
14	72	2.1	3.0	11.0	15	270	73	8.8	36
10	66	1.6	2.4	6.8	10	540	65	5.4	26
14	72	2.1	2.9	11.3	16	800	70	9.1	37
16	72	2.2	3.0	12.2	23	1070	70	13.1	54
10	79	1.8	2.3	7.6	10	1330	79	5.9	26
Body No. 6 (S)									
10	72	2.2	3.0	9.3	13	0	69	6.8	34
10	75	1.6	2.1	6.6	9	270	74	5.2	23
10	72	1.8	2.5	7.7	11	540	74	5.6	29
10	72	1.4	1.9	5.8	8	800	72	4.6	21
10	73	1.5	2.0	6.3	9	1070	72	4.6	23
10	72	0.9	1.2	3.7	5	1330	72	2.7	14

to the difference in the dispersion of the test data under the two investigations, or to the asymmetric state of the probable errors in each observation. If, as seems to be indicated by the nature of the breakdown of the various bodies, the compression specimens fail ultimately in tension there should be a far better agreement. Another series of specimens gave approximately the same results.

A comparison of the dielectric strength of the specimen with its mechanical strength yields for compression and tension specimens (A and E, B and F) correlations of 0.72 and 0.88 respectively, indicating more than merely a relationship. This leads to the natural conclusion that the better the material mechanically, the greater the dielectric strength of the body; this property is markedly brought out in later studies of the material under both mechanical and electrical stress.

In order to determine whether the breaking of the specimens under load at the time of puncture was related to either the dielectric or the mechanical properties of the material, column G was studied in conjunction with columns B and F, and the coefficients of correlation determined. In the case of B and G the correlation value was —0.05, a very unreliable indication that the stronger tension specimen would be more likely to fail at the time of puncture than the weaker specimen. The case of F and G gives a ratio 0.24, which also indicates a very uncertain relationship. The failure of the tension specimens at the time of puncture cannot be accounted for by either the particular dielectric or mechanical properties.

Much of the detail material has not been included in the discussion, but has been embodied in the tables, and an attempt has been made to furnish enough explanatory matter at the bottom of the tables to make them complete in themselves.

VI. Dielectric-Tension Tests

19. *Problem.*—The previous section dealt with the effect of puncture on the mechanical strength of the specimen and the interrelation of the properties of the specimens with no-load dielectric strength.

Another approach to the general problem is the investigation of the effect that loading of the specimen may have on the dielectric strength of the body. The first loading considered will be that of tension, with the voltage applied at right angles to the mechanical stress. This type of loading proved the most satisfactory from the standpoint

TABLE 11

STUDY OF MEAN AND STANDARD DEVIATION AND RANGE OF DIELECTRIC STRENGTH OF TENSION SPECIMENS

(Complete Series)

Body	Mean kv.	Median kv.	Mean Deviation kv.	Standard Deviation kv.	Range kv.	Coefficient of Variability per cent
			Mean Deviation			
K	10.1	11.5	3.4	3.8	12	38
L	10.3	9.8	1.5	2.2	7	21
M	6.5	7.8	2.0	2.1	6	33
N	10.4	11.4	2.0	2.2	6	21
P	8.5	8.7	1.9	2.5	8	30
S	4.9	4.9	1.0	1.2	4	25
			Standard Deviation			
K	12.2	13.4	4.4	4.8	14	40
L	12.6	11.4	2.4	3.1	9	24
M	8.5	9.2	2.3	2.7	9	32
N	13.0	14.2	2.8	3.0	8	24
P	9.3	9.3	2.2	2.3	6	24
S	6.6	6.5	1.3	1.7	6	26
			Range			
K	40.1	41.4	15.3	17.3	50	42
L	39.3	33.5	9.8	10.5	26	27
M	29.3	28.3	7.2	8.8	29	30
N	43.0	41.2	9.9	11.2	28	26
P	34.4	31.8	7.6	9.6	28	28
S	23.8	23.3	4.9	6.3	20	26

of observing the resultant phenomenon, for it is possible to load the specimen to the point of rupture without the electrodes being detached by the deformation of the specimen. In fact, it not infrequently happened that at the time of the electrical rupture the specimen ruptured simultaneously in tension.

20. *Test Data.*—As in the previous part of the investigation, the data have been tabulated in complete form which permits independent study. Table 10 gives a summary of the test made on the six bodies used in this investigation. The probable error of the mean is within three per cent or less. The data in the case of the tension specimens are not widely dispersed, as may be determined from the coefficient of variability. From the standpoint of reliability of the mean the conditions of the test would be considered satisfactory, and all specimens may be considered equally reliable for a comparative study.

TABLE 12

STUDY OF MEAN AND STANDARD DEVIATION AND RANGE OF DIELECTRIC STRENGTH OF TENSION SPECIMENS UNDER VARIOUS LOADS

Load lb.	Mean kv.	Median kv.	Mean Deviation kv.	Standard Deviation kv.	Range kv.	Coefficient of Variability per cent
			Mean Deviation			
0	11.0	10.7	3.3	3.4	8.7	31
270	8.6	9.0	2.5	2.9	7.6	34
540	5.4	5.5	1.3	1.6	4.8	30
800	7.7	8.7	2.0	2.1	5.7	28
1070	8.8	8.7	2.5	2.9	8 5	33
1330	8.4	8.8	2.8	3.5	11.0	39
1600	10.6	11.8	1.9	2.0	4.5	19
			Standard Deviation			
0	13.4	13.4	4.5	4.7	12.2	35
270	11.0	10.9	3.3	4.0	10.0	37
540	6.8	7.3	1.6	1.9	5.4	28
800	9.5	10.3	2.1	2.4	6.2	25
1070	10.2	10.2	2.4	2.9	8.6	28
1330	10.6	11.2	3.7	4.3	13.2	40
1600	12.3	13.4	2.0	2.2	5.0	18
			Range			
0	45.9	45.4	12.1	13.9	40	30
270	36.7	33.3	11.3	12.8	32	35
540	25.1	27.5	5.3	5.8	15	23
800	30.9	31.7	6.7	7.4	21	24
1070	35.7	30.7	11.9	13.1	32	37
1330	36.5	36.4	14.3	15.2	41	42
1600	36.8	40.7	5.7	6.0	13	16

21. *Analysis of Data.*—To study the data and establish the reliability of the results, the three most determining statistical factors, mean deviation, standard deviation, and range, for the complete series, have been tabulated in Table 11.

Of these three factors special attention should be given to the standard deviation, and the average of this quantity for the bodies at the various loadings. This average does not have a very large range, and indicates that comparisons of these bodies should be under equally-weighted test conditions. The standard deviation for the group of bodies tested shows a very narrow range, as does also the average of the mean deviation.

The fluctuation of results, at the various loads, may be studied by referring to Table 12. The standard deviations at the extreme loadings are higher than at the middle ranges of loading. This indi-

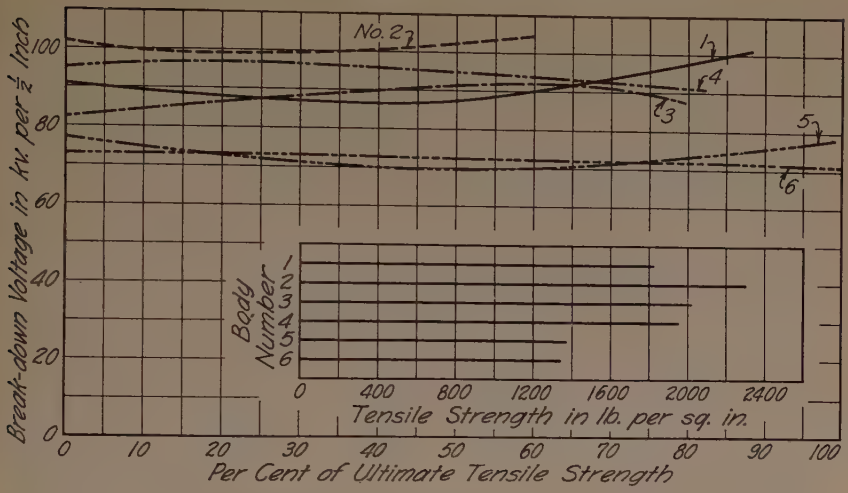


FIG. 6. RELATION BETWEEN DIELECTRIC STRENGTH AND APPLIED TENSILE LOAD

cates that there is more variation at the extremes of loading. This may have a physical explanation in the fact that at no load such fissures and faults as may exist are free and tend to assume their normal area, while as the load is applied they are distorted and their dimensions so changed as to reduce the areas. These areas again increase, as the load is increased, by the forming of destructive cracks and openings in the bodies.

The resultant smoothed averages obtained are plotted in a graphical form in Fig. 6. The characteristics are plotted against percentage of ultimate strength taken from the ratio of the actual load to the rupture load, as determined from the mechanical tests made previously on the specimens. Figure 6 also includes a small diagram giving the mechanical strength of the bodies, and shows the load and the dielectric strength of the various specimens taken from Table 10, a smoothing formula* having been applied. The resultant curve more nearly represents the definite law. The smoothing process need be applied only once to give very satisfactory results.

22. *Inference.*—A study of the curves fails to establish any definite law of performance of the material when tested under tension loading. Three of the bodies show first a decrease in the dielectric strength and then an increase, while the other three bodies show an opposite effect

*Let A, B, C, D, be the terms for points on the curve, smoothed values $A' = \frac{2A + B}{3}$, $B' = \frac{A + B + C}{3}$ etc.

or a variation dissimilar to that shown by the outstanding characteristic curves. Body No. 6 shows the least variation with the application of load; this body was manufactured under ideal laboratory conditions. Each body seems to follow a law of its own, which may be governed by the characteristic mix and burning of the body.

The whole case may be summed up by saying that there seems to be no fixed law governing the effect of the mechanical loading in tension upon the dielectric strength of the various bodies. Each body shows different characteristics of dielectric strength at different loadings, showing that the electrical strength of the body is modified by the loading. This influence is slight, and does not deviate very far from ten per cent as a maximum in the more extreme cases. The uniformity of the laboratory body indicates that control of the manufacture of the body might to some extent eliminate variation.

When attempting to analyze failure from a mechanical viewpoint, rather than from an electrical one, it would appear that continued increase of dielectric strength with loading, until very near the rupture point, or at least until the deformation becomes serious, represents the more probable condition. The material is undoubtedly laminated to a greater or less degree, depending on the body, but, since these laminations are in all directions, it is impossible to use failure along such laminations as a reliable basis of explanation for a change in dielectric strength over no-load condition. This leaves only fissures and flaws as a possible source of this change.

These types of imperfections in the material are in the nature of small voids, which in the case of a fairly homogeneous material are microscopic in size, while in the poorly-manufactured body they may consist of actual visible openings. When the material deforms under mechanical loading it is natural that it will follow the path of least resistance, therefore, moving into the voids in the body, with the result that the material for a time will elongate the voids, thereby increasing the electrical resistance, and causing an increase in the dielectric strength. However, if the loading is continued, the material begins to fail and the voids, due to the excessive stress, become points of local failure, thereby decreasing the resistance and allowing a flow of current through the material. The foregoing reasoning is based on the assumption that the controlling factor in the breakdown is the structure of the body, not the dielectric characteristics of an absolutely homogeneous body.

If acceptance is made of the general conception that the failure under compression, in the case of brittle materials, is in the nature of

a tension failure the foregoing consideration of the effects of flaws and fissures will satisfactorily apply to both the tension and the compression tests. However, when a study is made of the data presented in Table 9, it will be seen that, though there is some correlation between the compression and tension of porcelain, it is far from satisfactory, and cannot be classed as more than possible. It might be expected that the characteristics of the two tests made, considering dielectric strength, would not be the same.

That the tension bodies did not correspond needs some consideration if the reduction of the electric resistance is affected by the percentage of flaws and fissures. When a study is made of Fig. 6, bodies 1, 2, and 5 represent one type of characteristic, while bodies 3, 4, and 6 represent another. The dielectric strength of the first group of samples shows a marked tendency to first decrease with load and then increase as the load is increased, never falling off even when approaching closely to the mechanical rupture point. This group represents bodies from the complete range considered, the high group, the middle group, and the low group, and the behavior does not agree with the explanation given in the foregoing as the most desirable.

To form a definite and final conclusion on the number of bodies studied would no doubt lead to erroneous results, for it will be shown in the following discussion that the porcelain plays only a small part in the dielectric strength, but that the flaws and imperfections are the determining factors, the porcelain acting only as a supporting medium for a group of voids. Therefore there will be as many results as there are combinations and arrangements of internal structure.

VII. Dielectric-Compression Tests

23. *Plate Glass Tests.*—In connection with the study of the bodies under compression it was possible to make a preliminary study with plate glass. In the case of the tension specimens it was impossible to obtain plate glass samples that would permit of a similar study. The tests on the plate glass specimens gave some valuable information, which to some degree indicated the type of results that were being obtained in the test of the porcelain.

A large sheet of plate glass was cut into specimens measuring ¼ in. x 1½ in. in section and 3 in. in length, with the two ends ground parallel. These specimens were only ¼ in. thick, the thickness of the porcelain specimens being, as already noted, ½ in., but, since the dielectric strength of glass is higher than that of porcelain, the puncture

TABLE 13

SUMMARY OF DATA OBTAINED IN DIELECTRIC—COMPRESSION TESTS (SMOOTHED)

Body	Dielectric Strength kv.	Percentage Ultimate Strength	Body	Dielectric Strength kv.	Percentage Ultimate Strength
No. 1	88	0.0	No. 5	75	0.0
	94	4.8		79	6.0
	98	9.8		79	12.2
	97	14.7		77	18.2
	96	19.7		73	24.5
	93	24.6		70	30.4
	90	29.4			
No. 2	97	0.0	No. 6	79	0.0
	98	4.5		79	5.5
	98	9.2		76	11.2
	98	13.7		73	16.6
	96	18.3		69	22.3
	94	22.8		63	27.8
	92	27.3			
No. 3	86	0.0	Plate Glass	71	0.0
	92	5.5		85	12.0
	96	11.3		72	24.6
	93	16.8		63	36.6
	87	22.6		58	49.2
	82	28.1		56	61.1
No. 4	93	0.0			
	93	5.7			
	91	11.7			
	89	17.3			
	86	23.4			
	78	29.1			

The percentage ultimate strength is the percentage of the load that would rupture the specimen.

voltages were about equal in magnitude to those for the porcelain. These specimens were assembled with sealed-in electrodes and punctured under the test conditions established for the porcelain.

With the temperature constant and the voltage applied at a rate of one kilovolt per second, in a direction at right angles to the loading, the specimens were placed under several different mechanical loads and punctured. A summary of the data obtained is shown in Table 13 and the resulting curve is shown in Fig. 7. The dielectric strength at first increased to a maximum as the load was applied, and then decreased as the load was increased. There is no question as to the reliability of the results, for there is no difficulty in seeing the puncture and the condition of the glass before puncture. The specimens were free from any flaws, and in each breakdown there was a puncture present in the glass. The nature of these punctures and the characteristics of the punctures under various loads will be discussed in the next section.

In the investigation of the glass it was possible to load the specimen to sixty per cent of its ultimate strength without detaching the electrode. The presence of a loose electrode is easily detected, for the

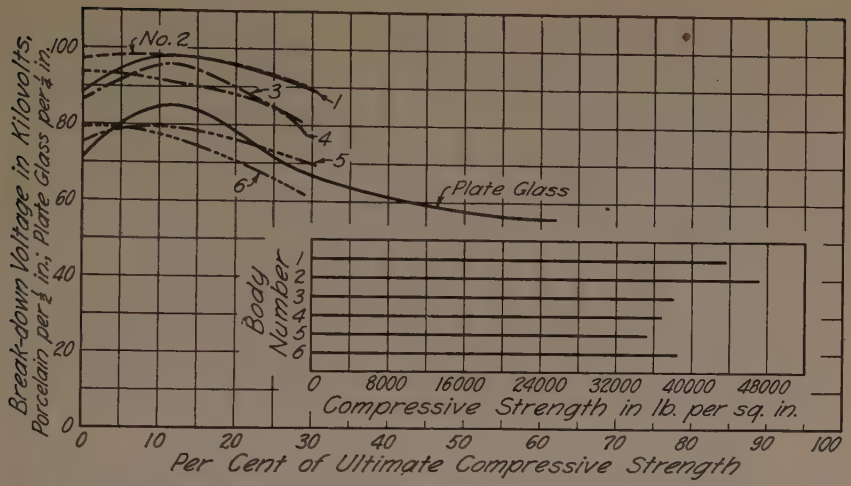


FIG. 7. RELATION BETWEEN DIELECTRIC STRENGTH AND APPLIED COMPRESSIVE LOAD

material will not puncture but will flashover. While it was possible to carry the plate glass up to a relatively high load, it was impossible to carry the compression tests of porcelain above thirty per cent of the ultimate load. The deformation and the character of the surface in the case of the porcelain caused the electrode to become loose at relatively light loads. It would be very desirable to obtain an electrode cement that would be elastic and at the same time have a high dielectric strength so that the compression specimens could be loaded mechanically, as in the case of the tension specimens, to a point where they would fail under mechanical stress at the time of the puncture.

24. *Porcelain Bodies.*—Table 13 shows the results obtained when testing the bodies under different loads. The probable error in each case has been held within three per cent, and the resultant curve smoothed.

An analysis of the porcelain test data shows that compressive loading first tends to increase dielectric strength up to a maximum of about ten per cent, and then the dielectric strength falls off to a value of about five per cent less than for no load, at the extreme loading possible with the present dielectric cements. The maximum dielectric strength occurs at between ten and fifteen per cent of the ultimate load.

25. *Inference.*—If it is accepted that the cracks and flaws play the more important part in affecting the dielectric strength of the ma-

terial, it will be easy to explain the phenomena represented by the curves by assuming that, as the load is applied, the fissures are closed, and the flaws are made more dense. When the body begins to deform under the loading, however, the fissures open and the material cracks, thereby reducing the dielectric strength. Referring to the previous discussion on the tension loading tests, the foregoing explanation of the effect of mechanical loading will be found to be more satisfactory in this case.

A reliable material, such as plate glass, which is under the observation of the experimenter both before and after puncture, obeys the general law. The plate glass is a very homogeneous material and not one full of visible flaws, but one with microscopic voids. There is also an agreement between the tension and compression tests, that corresponds well with the theory that all brittle material fails in tension even when the nature of the loading is such as to produce compression stresses.

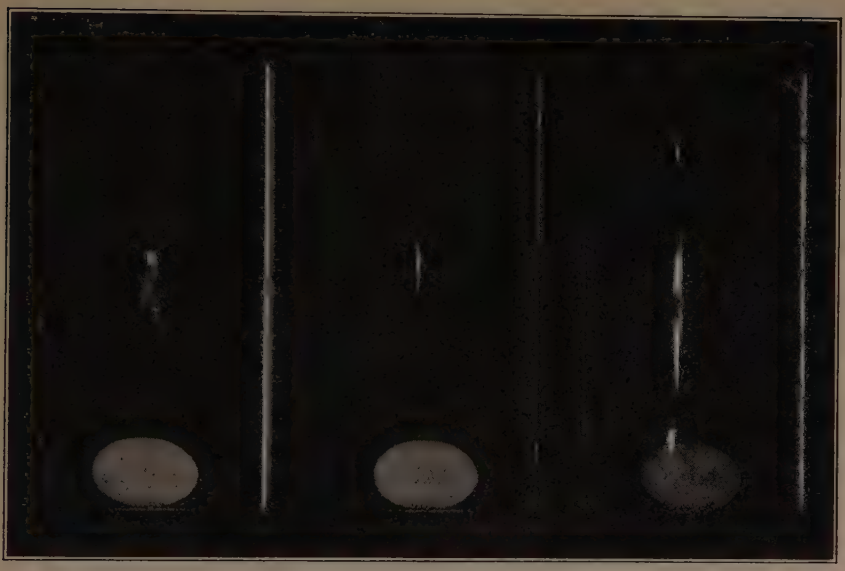
Even though there is some deviation in the case of the compression specimens the whole series of tests points very directly to the probable correlation between flaws and dielectric strength. The next section, which will deal with the nature of the punctures, will even more clearly substantiate this conception, for the nature of the puncture is directly traceable to weaknesses in the mechanical structure of the material.

It is obvious that the characteristics of the product of any manufacturer cannot be determined with certainty by the selection for test of only one run from each manufacturer, but such a selection will aid in obtaining generalized information as to the characteristics of manufactured bodies.

VIII. Nature of Puncture and Body

26. *Plate Glass.*—As in the case of the tests dealing with the effect of compressive stress on dielectric strength, it was thought advisable first to study the nature of the puncture in glass specimens. A puncture in glass gives a visible track that may be subjected to minute study without disturbing the surrounding unpunctured material, and the transparency of the material also makes the selection of flawless pieces possible.

Figure 8 shows a series of punctures made in glass specimens under various loadings in compression. It will be noted that, as the load was increased, the testing machine heads being applied to the ends



(a) Specimens set parallel to lens of camera

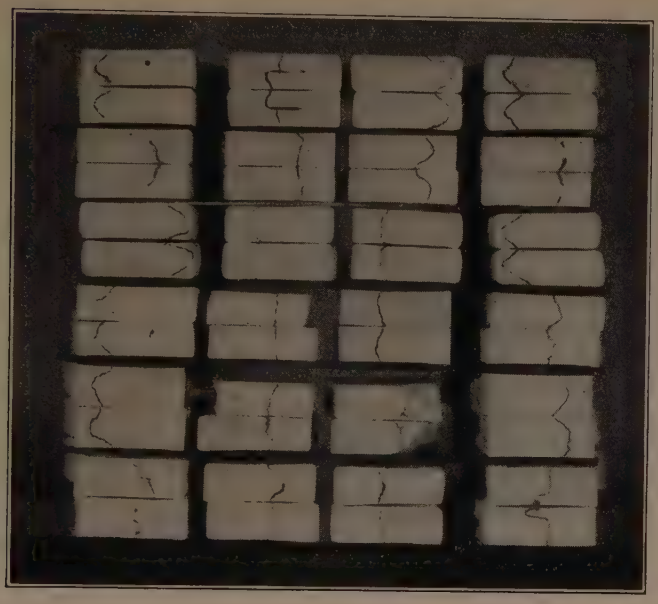
(b) Specimens set at an angle of 45 deg. to lens of camera

FIG. 8. PUNCTURE TRACKS IN PLATE GLASS SPECIMENS

of the specimens and the puncture being at right angles to the direction of loading, the puncture track was more and more confined to one plane. At zero load the track takes a spiral shape, and is surrounded throughout its length with fissures that project out into the glass. When the loading has been increased to twenty per cent of the ultimate strength of the glass the only fissures are in the direction of the loading, and the track lies very nearly in one plane, but as yet there is some bending of the track in the plane, and the path has considerable width. Increasing the load to forty per cent of the ultimate strength causes the fissures in the direction of the heads to increase, but the main puncture track is now very straight, and shows no tendency to project beyond the plane. The splitting of the specimen might be expected as all the movement in the direction of the loading is restrained while the sides are free, but this does not explain why the puncture should confine itself to this plane. The question may be asked as to whether or not the loading produced the fissure through which the current flowed. All specimens tested were subjected to loads equal to sixty per cent of the ultimate strength and examined before the electrodes were sealed on. At the higher loadings, forty percent of the ultimate strength and more, it was not unusual to have the specimen split through the entire length at the time of the puncture. To observe if the fissure formed before the puncture took place, the voltage on several specimens was increased one kilovolt at a time and then cut off and the specimen examined. In no instance was a fissure apparent, but in two instances the specimen was found to be partially punctured, showing some penetration without failure.

The lower view of Fig. 8 was taken with the specimen set at an angle of 45 deg. to the lens of the camera in order to set the puncture tracks in relief and it will be observed that the punctures of larger diameter are at zero load and at a load equal to twenty per cent of the ultimate strength, and the thread-like puncture at forty per cent of the ultimate load. These specimens were selected as representative of the group tested.

27. *Porcelain.*—It would be desirable to follow the same procedure for the porcelain specimens, but as the material was opaque the puncture tract was not directly visible, and, therefore, much must be left to the imagination. The x-ray was used in an attempt to study the track, but as the path was closed with fused glass and not enough of the metal carried into it to show on the plate, it was impossible to obtain the trace of the path without destroying the specimen.



A B C D

FIG. 9. PUNCTURE TRACKS IN PORCELAIN SPECIMENS

The two outer columns, A and D, are tension specimens and the two inner columns, B and C, are compression specimens

There are available two methods of procedure, one, breaking the specimen along the track and noting the course traveled by the current, and the other, taking sections through the puncture, and examining the specimen by means of a microscope. Both methods were resorted to in an attempt to determine the normal path of least resistance.

Figure 9 shows a group of selected specimens that give the characteristic type of puncture observed when the specimens are broken open at the track.

The specimens broke rather easily at the puncture by merely holding them in the hand and striking them with a round rod. The tension specimens, being longer than the compression specimens, broke more readily. The two outer columns are the tension specimens and the two inner columns are the compression specimens. There seems to be no marked difference between the punctures of the two types of test specimens. Another feature to be noted is that, though the fused glass did not show well under the x-ray, the contamination of the fused material was sufficient to cause a discoloration which permits

TABLE 14

DIELECTRIC STRENGTH OF PUNCTURE TRACK SPECIMENS SHOWN IN FIG. 9

Body	Dielectric Strength (in kv.) of Specimens in Columns			
	A	B	C	D
1	92.4	95.5	81.6	79.1
2	82.2	105.2	79.9	106.8
3	93.5	102.5	99.0	93.0
4	88.9	99.4	112.7	100.8
5	87.4	105.2	111.9	84.4
6	68.9	97.1	97.9	74.8

The puncture tracks have been classified according to the following system:

I. Straight puncture path
II. Slightly curved puncture path
III. Definitely curved puncture path
IV. Scattered puncture path

The average breakdown voltage corresponding to the foregoing classification was

I. 102.8 kv.
II. 95.8 kv.
III. 88.0 kv.
IV. 67.0 kv.

The correlation coefficient between the track and the breakdown voltage is 0.68.

the eye to follow the track, and by using plates and paper giving maximum contrast it is possible to obtain satisfactory photographs.

The outstanding feature is the relation between the nature of the puncture track and the breakdown voltage. A direct path signifies a high electric stress before specimen failure, while both more and longer paths occurred at low breakdown voltages. When the paths are of a multiple nature, as shown by the specimen designated as A6, there is the least electrical strength. This occurs very consistently, as in no instance was a path of this type found with any but the lowest breakdown voltages.

If the specimens are observed very closely it will be seen that the puncture track frequently centers around some type of flaw in the material. In the straight punctures this is not true, for in all these cases the material in the puncture region seemed to be the same as the rest of the specimen except in the puncture itself. The flaws show up much more clearly in the actual specimen than in the photographs. Specimen B1 shows a flaw in the very center of the piece, but this does not necessarily cause the lowest breakdown voltage; several pieces of this type gave good results as compared with solid bodies. It may be noted that this type of flaw appeared only in the compression specimens.

Table 14 lists the breakdown voltages of the various specimens in

the order of the arrangement in Fig. 9. Dividing the specimens into four groups with characteristic punctures as follows: straight, slightly curved, pronouncedly curved, and scattered, it is possible to arrive at a correlation between the puncture track and the breakdown voltage of the specimen. In the study of the correlation the two specimens 1B and 5C were thrown out, these having been included in the group shown in Fig. 9 only to show a flaw and a forked type of puncture. The flaw type did not occur often enough to justify a special classification, and the forked type should be classified in the group characterized as having the major part of the path outside of the fork, in this case in the straight classification. The correlation for the group shown in Fig. 9 is 0.68 and positive, with the straight punctures having high breakdown voltages and the scattered punctures having low breakdown voltages. When the correlation was determined for a large group of specimens the coefficient was 0.76.

28. *Microscopic Study.*—Turning to the microscopic investigation of the punctures, Figs. 10, 11, 12, and 13 show representative studies of the various types of possible puncture sections. The four views of Fig. 10 show actual sections, magnified to eighty diameters, of specimens that typify the general classes of punctures. The punctures may be classified into three general types, those that are scattered, covering a large area and indicating no very definite path, those that are concentrated, and in general have a path of circular cross-section which may vary in diameter from the size of quartz grains to twenty mils, and a mid-classification which may be from two to eight times as long as it is wide, with a very definite opening.

Figure 10(a) shows a section taken along the puncture path in which the composition of the path may be studied. The path is congested with fused glass, which is the by-product of the intense heat generated by the passing of the current at the time of breakdown. This formation of a glass matrix in the puncture track accounts for the fact that it is impossible to draw a lead solution into the hole by means of a vacuum in order that an x-ray of the path might be taken. The figure shows how the path is completely filled with glass after the puncture. Others show small formations of glass surrounding the puncture hole. Figure 10(d) is a view taken through the glass formation in the track; the glass appeared to be discolored to a reddish brown, which no doubt was due to the metal carried into the track from the electrodes at breakdown.

In the eight specimens given on Figs. 11 and 12, there have been

arranged the types of punctures that occurred under the various loadings in tension and compression. It will be observed that one type of puncture does not confine itself to one type of loading, as it is possible to find the punctures of the various classes distributed among all the loadings. Diagrams are given of some of the punctures selected at random. Since any class of puncture may be found under load with any type of loading, the natural inference is that the character of the puncture is not influenced by the load.

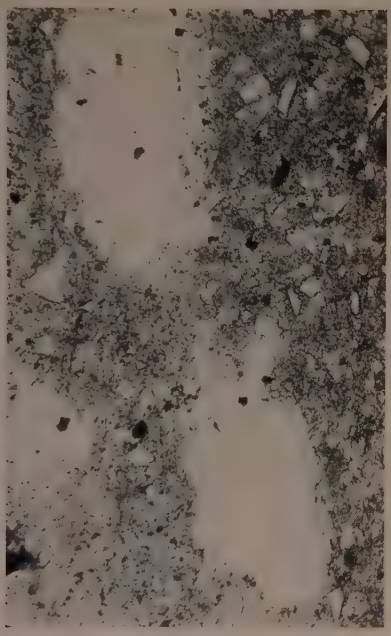
Figure 13 shows two types of punctures selected at random from specimens of high dielectric strength and two from specimens of low dielectric strength. As these have been arranged it will be seen that the different types of punctures are found with both the high- and the low-dielectric-strength specimens, and as in the previous case there cannot be shown the least relationship between the type of puncture and the breakdown voltage as indicated from the sections. The coefficient of correlation of the round puncture to the high breakdown voltage as calculated from the specimens sectioned is 0.05, indicating a lack of any relationship between the type of the puncture and the breakdown voltage.

These studies of puncture track under the microscope are naturally somewhat incomplete, as the number of specimens is necessarily limited, since it is expensive to obtain the sections. The deductions drawn have been obtained from 42 sectioned samples. Any statistical study of relationship must be very uncertain unless guided by experience in the study of the specimens. There is also to be considered the fact that the same puncture track may show a variety of sections, depending on the point at which the section is taken. It is possible to take only one section of a single puncture, for the remainder of the material must be ground away. The conclusion that there appears to be no definite relationship between the puncture and the breakdown voltage, as revealed by a study of sections, is verified both by a statistical study and by actual examination of the sections. The samples selected for the illustrations were random selections from the sections available so that the nature of the distribution might be clearly recorded.

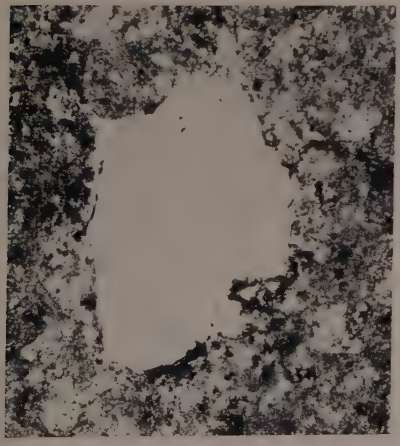
The microscopic investigation has been extended to a study of the nature of the porcelain body* in an effort to find some reason for the differences in the breakdown voltages of the material. Table 15 is a study of the structure of the material in conjunction with the average

*The sections were made in the Department of Ceramic Engineering and the studies summarized in Table 15 were made by Mr. A. J. Monack. The column giving dielectric strengths was taken from previous studies of the bodies.

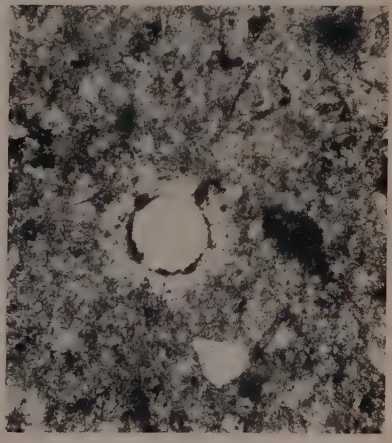
(a) Longitudinal section of puncture path



(b) Section of scattered puncture path

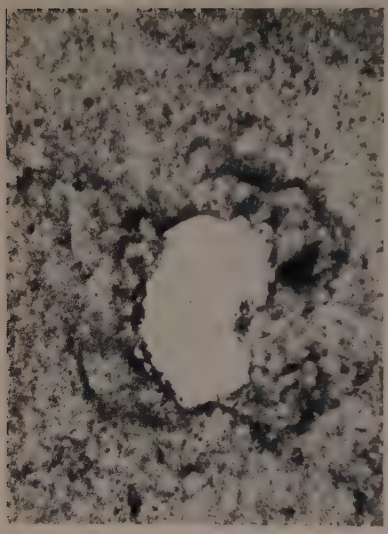


(c) Puncture path reduced to one path

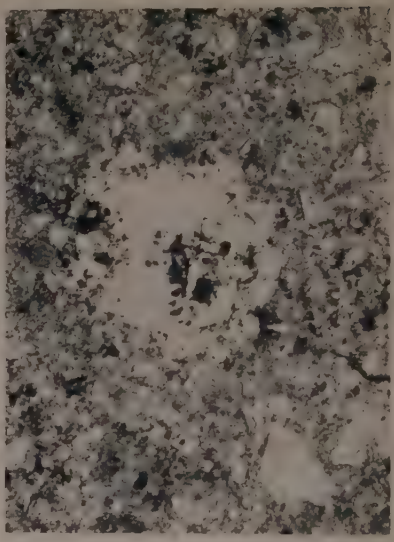


(d) Puncture path concentrated to hole of circular cross-section

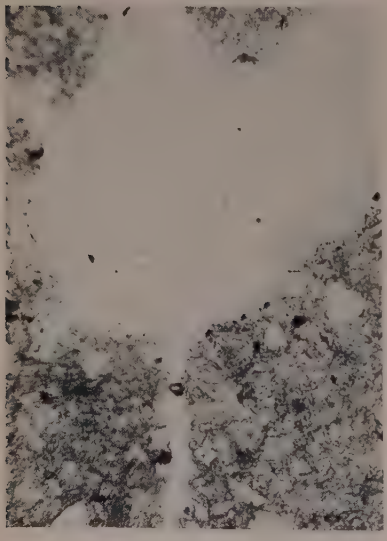
FIG. 10. MICROSCOPIC STUDY OF PUNCTURE PATHS



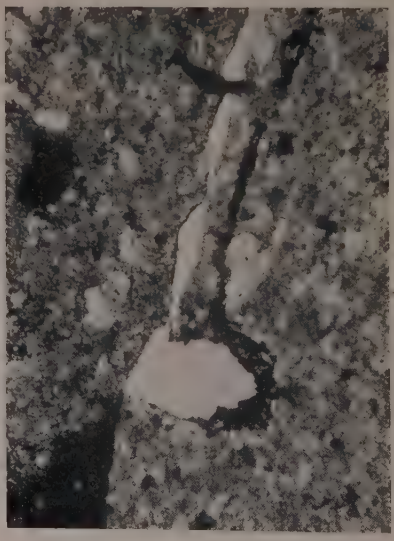
(a) Zero loading



(b) Loading of 540 lb. per sq. in.



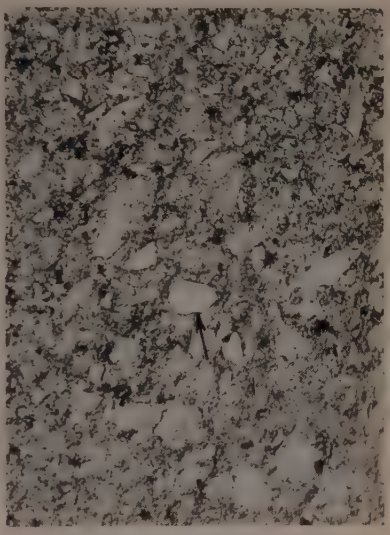
(c) Loading of 1070 lb. per sq. in.



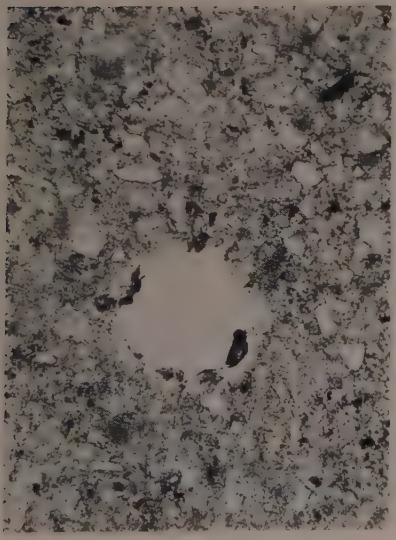
(d) Loading of 1200 lb. per sq. in.

FIG. 11. VARIATION OF NATURE OF PUNCTURE WITH LOADING—TENSILE LOADS

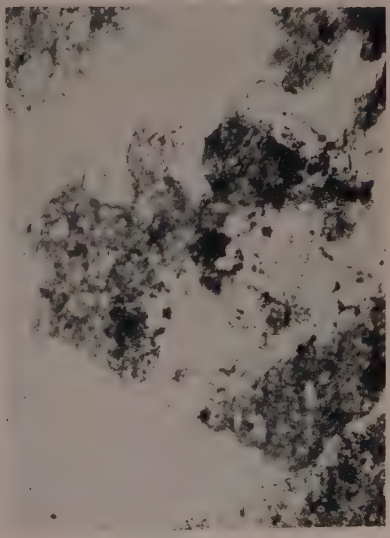
(e) Zero loading



(f) Loading of 4300 lb. per sq. in.

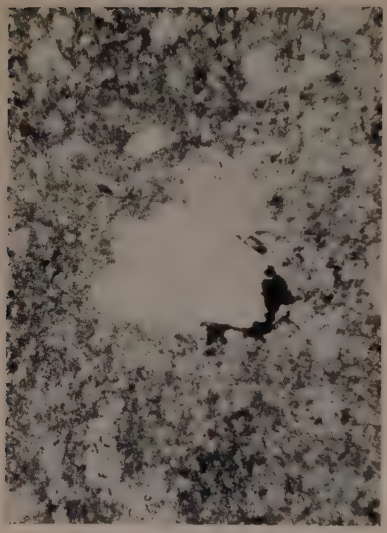


(g) Loading of 8600 lb. per sq. in.

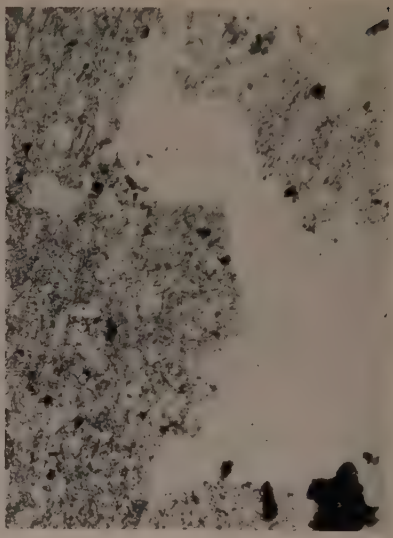


(h) Loading of 12 800 lb. per sq. in.

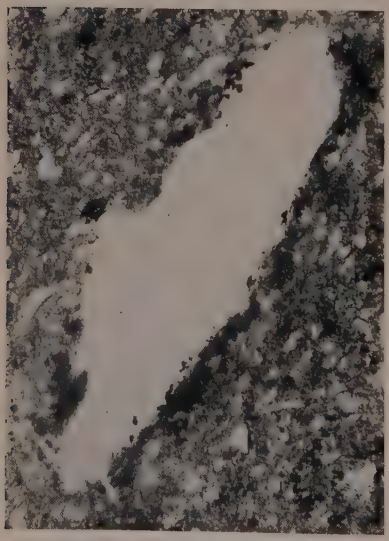
FIG. 12. VARIATION OF NATURE OF PUNCTURE WITH LOADING—COMPRESSIVE LOADS



(a) 102.5 kv. per ½ inch



(b) 117 kv. per ½ inch



(c) 78 kv. per ½ inch

(d) 74 kv. per ½ inch

FIG. 13. VARIATION OF NATURE OF PUNCTURE WITH BREAKDOWN VOLTAGE

TABLE 15

RESULTS OF MICROSCOPIC EXAMINATION OF PORCELAIN SPECIMEN SECTIONS

Body	QUARTZ			Mullite Development	Pores	Micro-structure	Dielectric Strength
	Solution	Size	Distribution				
1	Very little	Small	Good	Fairly good	Not many	Good	89
2	Very little	Small	Good	Fair	Numer-ous	Good	97
3	Some	Small	Good	Fair	Some Small	Fair	85
4	Little	Medium	Good	Good	Not many	Good	93
5	Little	Small	Excellent	Fair	Rela-tively Large	Good	77
6	Little	Rela-tive Large	Good	Good	Some	Fair	76

breakdown voltage for the body being studied. This table, when carefully scrutinized, does not show any relationship between the structure and the breakdown voltage. There is not a single item that would account for the difference in breakdown voltage between No. 2 and No. 6, the highest and lowest. The porous nature of No. 2 would lead to the initial assumption that the material was very poor, but this does not prove to be true. The reason is that this porous structure is only relative and not of major importance. It is not possible to determine the difference in per cent porosity between the two bodies by normal test methods. If the porosity was of the nature present in under- and over-burned bodies, a few openings would approximately cover the field of the microscope. The conditions shown by Table 15 may indicate that after the pores have been reduced to a certain size porosity is not the determining factor in the dielectric strength of a body as compared with the other factors, doubtless including flaws and manufacturing imperfections.

IX. Summary

29. *Summary of Conclusions.*—The results of the investigation of porcelain bodies may be summed up in the following items:

(1) A solid sealed-in electrode is satisfactory for dielectric tests, and compares favorably with the mercury electrode recommended by the A.S.T.M. The sealed-in electrode gives consistent results, and can be rapidly prepared and assembled in large quantities.

(2) A study of the dielectric strength of the porcelain from the mean value alone is of limited value, for it does not give a true interpretation of the quality of the material. A study of this mean value must be accompanied by a study of the variability of the material to have any definite meaning.

(3) A punctured porcelain body retains approximately eighty-five per cent of its original strength when subjected to compression loading.

(4) A punctured porcelain body retains approximately seventy-five per cent of its original strength when subjected to tension loading.

(5) The micro-structure of the body does not indicate its dielectric strength, for neither mullite development nor the solution, size, or distribution of the quartz crystals is correlated with the breakdown voltage.

(6) In the study of the series of bodies used in this investigation there was found a definite tendency for the bodies to have the same rank number whether the test was for compression, tension or dielectric

strength. This indicates that superior quality in one property will probably be accompanied by superiority in the others.

(7) There is a marked relationship between the mechanical and the electrical strength of the bodies. The correlation between compressive and dielectric strength is 0.71, while that between tensile and dielectric strength is 0.88.

(8) When the bodies are subjected to electrical stress while they are under a tension load there is some change in the breakdown voltage. The results obtained do not indicate that there is a specific law governing these conditions for each specimen seems to follow its own characteristic behavior. In general, it may be stated that the major portion of the specimens pass through a reversal of behavior as the mechanical load is applied. These dielectric-tension tests cover a range large enough and include a sufficient number of specimens to make the results reliable.

(9) The size of the sample used in the determination of the dielectric strength of the porcelain body must be large in order that the results may be reliable, necessitating between twenty and thirty specimens when relying on averages.

(10) When the bodies are subjected to electrical stress while they are under a compression load there is a marked and definite law of performance. The breakdown voltage, from zero load to the maximum load studied, steadily increases until about fifteen per cent of the ultimate load strength is reached, when there is a reversal, and the breakdown voltage falls off. The number of specimens studied up to the last loading considered was sufficient to give reliable results, but it was impossible to carry the investigation above thirty per cent of the ultimate strength, as the electrodes were loosened by the distortion of the body due to the stress.

(11) A study of the tests through the zero load point does not show any definite or continuous correlation between the tension and compression strengths.

(12) A macroscopic study of the puncture track definitely indicates the electrical strength of the material. Straight puncture paths indicate high dielectric strength, while the curved and scattered punctures indicate correspondingly lower dielectric strength.

(13) A microscopic study of the puncture track gives no indication of the electrical strength of the material, as any type of puncture may occur with any type of body as represented by the products of the various manufacturers. The specimens that gave both high and low

breakdown voltages for any one body showed similar types of punctures when studied in sections.

(14) The section of the puncture track does not indicate the character or the amount of the mechanical load that was applied to the specimen. Any load, either in tension or compression, may give the same type of characteristic puncture section.

(15) The lowered dielectric strength of the porcelain body under load is not a characteristic of any one particular body, but is similar for the various manufactured and laboratory bodies. There is no marked agreement between the degree of weakening in the bodies subjected to tension and compression, respectively, i.e., those bodies which weaken most under compression do not weaken most under tension. The correlation for this tendency is — 0.34, which shows that the tendency is present without being very marked. The fact that the test specimens are not of approved standard shapes may influence the correlation of results.

(16) The microscopic porosity of the material does not indicate the dielectric strength of the material when properly burned. Burning to excessively low and high cones gives very porous bodies with low dielectric strength but in the region of normal burning the porosity curve and the dielectric curve are both flat. The association of porous bodies with high dielectric strength is merely relative in the microscopic investigations. Those bodies which show pores under the microscope would fail to indicate any porosity under the normal approved tests.

(17) A study of the dielectric strength of the available porcelain is the study of the chance of the presence of flaws. The breakdown, in the major portion of the instances, is through some defect in the material which is apparent when the body is opened at the puncture track. That this flaw or defect is not produced by the current passing through the section, or by the intense heat at that time, may be concluded as a result of a study of the puncture track in glass, which produced a rather limited effect at no load, and a very thread-like puncture when the specimen was loaded. At the present state of development of a homogeneous porcelain it may be stated that the dielectric test is merely a check on the manufacturing processes, and not the electric strength of the material. A comparatively perfect porcelain should give many times the dielectric strength of the products at present available.

APPENDIX A

Statistical Methods

1. *Presentation of Data.*—The presentation of data by means of statistical methods in preference to the normal method of central tendency, called the average, has been made use of in this investigation on porcelain bodies. The principles underlying the details will be outlined in this section and, in order to eliminate confusion, the factors which have been generally recognized in a number of scientific fields will be presented without the mathematical discussions associated with them in the normal treatment of statistics.

The discussion of these methods will be confined to comparatively small samples of less than thirty specimens. The small sample is, of course, the more doubtful field from which to generalize, but a considerable amount of work has been directed to this field in recent years. It stands to reason that statistical methods must be used with judgment by the engineer. With a limited number of observations, considerable knowledge is necessary concerning the small sample.

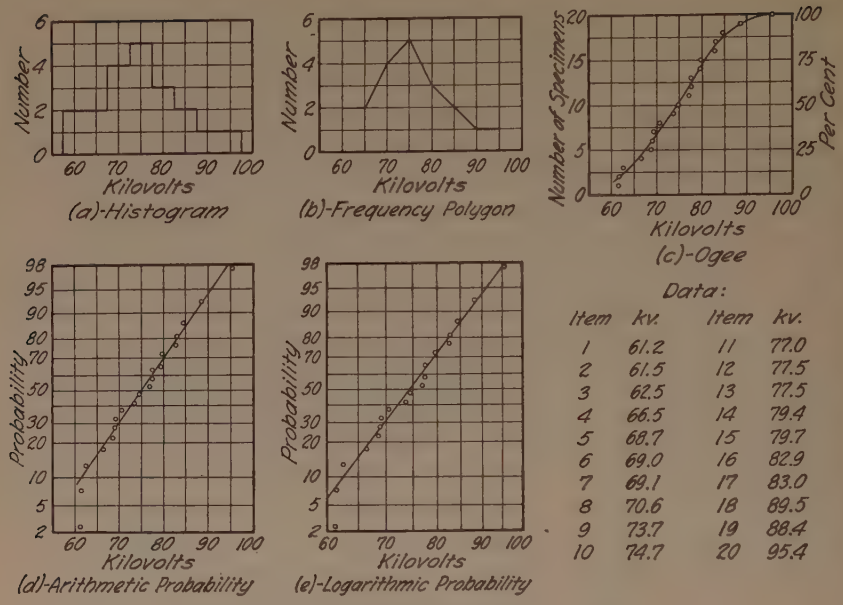
The first important question is the presentation of the data in a form permitting rapid generalization and comparison. Figure 14 shows five different methods of graphic presentation of data obtained in making the dielectric test on a porcelain body.

(a) The Histogram, or Column Diagram

Here the interval is represented by means of a straight line erected on the midpoints of the class intervals. The areas of the rectangles are assumed to represent the measures, and it is also assumed that the measures are distributed uniformly in the interval. This method of graphical record is not suitable for a small number of specimens, as it is necessary to group the data taken into class groups, and therefore the unusual or extreme values tend to hold a predominating place in the picture. The one factor to which it does give prominence is the *mode,* or the most common value. Since the normal dielectric test deals with thirty specimens or less the histogram must be essentially a crude device in this case.

(b) The Frequency Polygon

Here the mid-values of the intervals are joined by straight lines, and the area under the polygon represents the total number of measures. This form of presentation holds its place in the statistical field due to its approximation to the normal law of error. This method



Data:

Item	kv.	Item	kv.
1	61.2	11	77.0
2	61.5	12	77.5
3	62.5	13	77.5
4	66.5	14	79.4
5	68.7	15	79.7
6	69.0	16	82.9
7	69.1	17	83.0
8	70.6	18	89.5
9	73.7	19	88.4
10	74.7	20	95.4

FIG. 14. GRAPHIC PRESENTATION OF DATA

applied in the case of the small sample gives distorted conceptions, and is useful only in indicating the mode value.

(c) Ogee or Ogive

The two foregoing methods are quite frequently the only ones used, and have been given for this particular reason so as to compare them with the three following, which in themselves give a more accurate interpretation of data. The ogee or ogive is a cumulative curve obtained by plotting the mid-points of groups or the individual observations, arranged in ascending order of magnitude. Referring to *c*, Fig. 14, in which observation 1 is 61.2 kv. and observation 2 is 61.5 kv., cumulative plotting will place the first value opposite 1 and the second value opposite 2, and so on until the 95.4 kv. of the last specimen is plotted opposite 20. The diagram at once shows what number of specimens are below or above any point of breakdown. If the ordinates are marked in per cent the same information may be determined directly in per cent. The information plotted is for the sample tested, and therefore only for the limited number presented. Each group of twenty specimens will have an independent curve more or less like the first. In order to determine the condition for the complete "universe" the

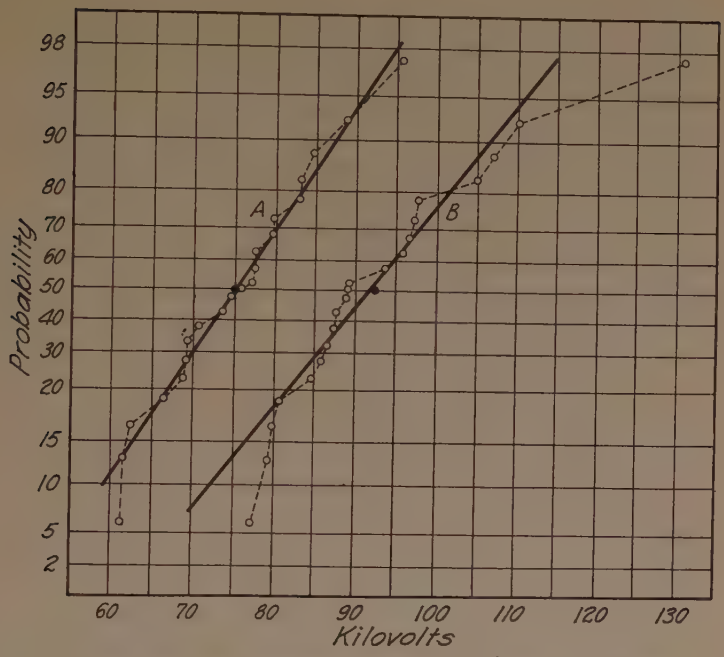


FIG. 15. DATA FOR DIELECTRIC STRENGTH OF PORCELAIN PLOTTED ON ARITHMETIC PROBABILITY PAPER

normal curve may be drawn through the mean and the data read with the same coördinates.

(d) Arithmetic Probability Plotting

When the data are plotted on probability paper the normal law can be readily presented by means of a straight line. The method lends itself well to small samples, and facilitates the approximation of the normal case. This type of paper bears a relation to cross-section paper similar to that the logarithmic papers bear, making possible the plotting of a curve of higher degree as a straight line.

This paper is adaptable to plotting both group or individual observations. In the case of the small sample the observations are plotted as individual points, more or less irregular in their arrangement, which may be replaced by a straight line. Figure 15 shows such a plotting of data for two porcelains tested for dielectric strength. The data for each of the samples consist of twenty observations of punctured specimens. The sample *A* is from the data as given on Fig. 14(d).

TABLE 16

DATA FOR DIELECTRIC STRENGTH OF PORCELAIN
CURVE A, FIG. 16

Observation	Dielectric Strength kv.	Probability per cent
1	61.2	2.5
2	61.5	7.5
3	62.5	12.5
4	66.5	17.5
5	68.7	22.5
6	69.0	27.5
7	69.1	32.5
8	70.6	37.5
9	73.7	42.5
10	74.7	47.5
11	77.0	52.5
12	77.5	57.5
13	77.5	62.5
14	79.4	67.5
15	79.7	72.5
16	82.9	77.5
17	83.0	82.5
18	84.5	87.5
19	88.4	92.5
20	95.4	97.5

The data must first be prepared for plotting on probability paper. Table 16 shows the prepared data for curve *A*. The first column is merely the number of the group or observation as listed. It will be noted that the dielectric strengths are listed in order of increasing magnitude; either increasing or decreasing magnitude may be used with the same result. The table as arranged will cause the origin to fall toward the lower left hand side of the coördinate paper. The last column headed "Probability" gives the percentage of the total observations presented by each item in a cumulative sense. The percentage for any item may be found by the following expression:

$$\text{Percentage} = \frac{2n-1}{2N}$$

where n is the observation number and N is the total number of observations. Though this causes the two end terms to be slightly in error, it proves a satisfactory method for plotting data and the terms need not be considered in drawing the straight line representing the universe, of which the data are a random sample.

Table 17 gives the percentage probability (last column Table 16) for samples from four to thirty, which would cover most porcelain experiments.

The following is the type of information which may be read directly from Fig. 15, *A:*

(a) The median, the mean and the mode all fall at 75.2 kv. since the straight line represents a normal distribution. This value is influenced by the most likely position of the straight line.

(b) One in 10 specimens will fall below 59 kv., and one in ten will be above 87.5 kv. By extending the straight line it is possible to predict the number per 100 or per 10 000 that will fall within definite limits.

(c) Thirty per cent of the specimens will fall below 70 kv.; that is, approximately one out of three.

(d) Fifty per cent of the specimens will fall between 68 and 83 kv.

(e) Ninety per cent of the specimens will fall between 59 and 87.5 kv.

This additional information is obtained from the same data as are used to determine the mean value.

Probability paper is one of the most satisfactory means of plotting data in a form that will limit the drawing of ridiculous conclusions from a central tendency. This does not, however, rank with the use of specific information that is always available to the experimenter on any problem studied.

The slope of the lines *A* and *B* indicates the variability of the two samples. The conclusion is that *B* is a more variable body than *A*, and when the data are subjected to the usual mathematical analysis *A* has a coefficient of 0.12 and *B* a coefficient of 0.15, not a marked difference, but the difference in slope can be recognized.

It is also possible to draw the line to a percentage base where some value such as the mean or median is considered the base, and the other values are expressed as a percentage of this base.

(e) Logarithmic Probability Paper

This type of paper is used where the data have considerable skew. In the testing of the dielectric strength of porcelain the data give a close enough approximation to a straight line on arithmetic paper.

Of the preceding five methods (c) and (d) are the most useful, but it is well to keep in mind method (e) for use where special difficulty is encountered.

2. *Statistical Terms and Their Meaning.*—Each statistical term has a mission of its own in making clear the data obtained and in correlating the specific data with what may be expected if all the material of the same class were investigated under the same conditions. It is true that when data have been analyzed by statistical methods there

Table 17

Coefficients for Plotting on Probability Paper; Samples from 4 to 30

	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	
1	1.67	1.72	1.79	1.85	1.92	2.00	2.08	2.17	2.27	2.38	2.50	2.63	2.78	2.94	3.13	3.33	
2	5.00	5.17	5.36	5.56	5.77	6.00	6.25	6.52	6.82	7.14	7.50	7.89	8.33	8.82	9.38	10.00	
3	8.33	8.62	8.93	9.26	9.62	10.00	10.42	10.87	11.36	11.91	12.50	13.16	13.89	14.71	15.63	16.67	
4	11.67	12.09	12.50	12.96	13.46	14.00	14.58	15.22	15.91	16.67	17.50	18.42	19.44	20.59	21.88	23.33	
5	15.00	15.52	16.07	16.67	17.31	18.00	18.75	19.57	20.45	21.43	22.50	23.68	25.00	26.47	28.13	30.00	
6	18.33	18.97	19.64	20.37	21.15	22.00	22.92	23.91	25.00	26.19	27.50	28.95	30.56	32.35	34.38	36.67	
7	21.67	22.41	23.21	24.07	25.00	26.00	27.08	28.26	29.55	30.95	32.50	34.21	36.11	38.24	40.63	43.33	
8	25.00	25.86	26.79	27.78	28.85	30.00	31.25	32.61	34.09	35.71	37.50	39.47	41.67	44.12	46.88	50.00	
9	28.33	29.31	30.36	31.48	32.69	34.00	35.42	36.96	38.64	40.48	42.50	44.74	47.22	50.00	53.13	56.67	
10	31.67	32.76	33.93	35.19	36.54	38.00	39.58	41.30	43.18	45.24	47.50	50.00	52.78	55.88	59.38	63.33	
11	35.00	36.21	37.50	38.89	40.38	42.00	43.75	45.65	47.73	50.00	52.50	55.26	58.33	61.77	65.63	70.00	
12	38.33	39.66	41.07	42.59	44.23	46.00	47.92	50.00	52.27	54.76	57.50	60.53	63.88	67.65	71.88	76.67	
13	41.67	43.10	44.64	46.30	48.08	50.00	52.08	54.35	56.82	59.52	62.50	65.79	69.44	73.53	78.13	83.33	
14	45.00	46.55	48.21	50.00	51.92	54.00	56.25	58.70	61.36	64.29	67.50	71.05	75.00	79.41	84.38	90.00	
15	48.33	50.00	51.79	53.70	55.77	58.00	60.42	63.04	65.91	69.05	72.50	76.32	80.56	85.29	90.63	96.67	
16	51.67	53.45	55.36	57.41	59.62	62.00	64.58	67.39	70.45	73.81	77.50	81.59	86.11	91.18	96.88		
17	55.00	56.90	58.93	61.11	63.46	66.00	68.75	71.74	75.00	78.57	82.50	86.84	91.67	97.06			
18	58.33	60.34	62.50	64.81	67.31	70.00	72.92	76.09	79.55	83.33	87.50	92.11	97.22				
19	61.67	63.79	66.07	68.52	71.15	74.00	77.08	80.43	84.09	88.10	92.50	97.37				96.43	14
20	65.00	67.24	69.64	72.22	75.00	78.00	81.25	84.78	88.64	92.86	97.50				96.15	89.29	13
21	68.33	70.69	73.21	75.93	78.85	82.00	85.42	89.13	93.18	97.62				95.83	88.46	82.14	12
22	71.67	74.14	76.79	79.63	82.69	86.00	89.58	93.48	97.73				95.46	87.50	80.77	75.00	11
23	75.00	77.59	80.36	83.33	86.54	90.00	93.75	97.83				95.00	86.36	79.17	73.08	67.86	10
24	78.33	81.03	83.93	87.04	90.38	94.00	97.92				94.44	85.00	77.27	70.83	65.38	60.71	9
25	81.67	84.48	87.50	90.74	94.23	98.00				93.75	83.33	75.00	68.18	62.50	57.69	53.57	8
26	85.00	87.93	91.07	94.44	98.08				92.86	81.25	72.22	65.00	59.09	54.17	50.00	46.43	7
27	88.33	91.38	94.64	98.15				91.67	78.57	68.75	61.11	55.00	50.00	45.83	42.31	39.29	6
28	91.67	94.83	98.21				90.00	75.00	64.29	56.25	50.00	45.00	40.91	37.50	34.62	32.14	5
29	95.00	98.28				12.50	70.00	58.33	50.00	43.75	38.89	35.00	31.82	29.17	26.92	25.00	4
30	98.33					37.50	50.00	42.67	35.71	31.25	27.78	25.00	22.73	20.83	19.23	17.86	3
						62.50	30.00	25.00	21.43	18.75	16.67	15.00	13.64	12.50	11.54	10.71	2
						87.51	10.00	8.33	7.14	6.25	5.56	5.00	4.55	4.17	3.85	3.57	1
						4	5	6	7	8	9	10	11	12	13	14	

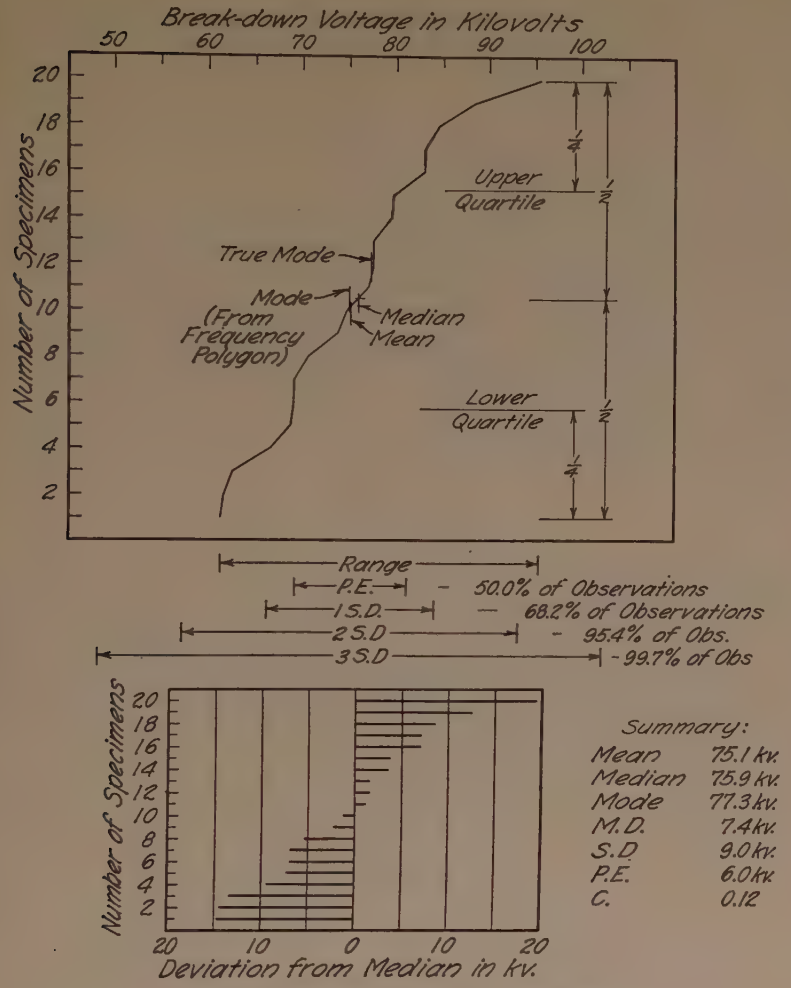


FIG. 16. STATISTICAL TERMS ILLUSTRATED

is less tendency to be unduly enthusiastic about drawing definite conclusions. Figure 16, by means of graphical interpretation, attempts to set forth the more common forms of descriptive terms used.

The *Mode*, which has been already mentioned, is the most commonly occurring value. In the case of the small sample it has no definite value, for, when creating class intervals, the value is merely an approximation. The mode is considered satisfactory when the "random sample" is large enough; this occurs when an added equal number of observations does not appreciably change the average.

A more satisfactory way of obtaining the mode is by means of the following formula:

$$\text{Mode} = \text{Mean} - 3\ (\text{Mean} - \text{Median})$$

which, for the *theoretical mode* gives 77.3 as compared with 75 determined from the grouping of the data.

The *Median* is the point in the distribution on each side of which half of the observations fall. Where there are an odd number of observations the median is the middle one, and where there are an even number, the average of the middle two terms is sufficiently accurate.

The *Average* or *Mean* has from the earliest times been considered as the most likely value for any set of observations. It is obtained from the summation of the observations divided by the number of observations, and is expressed by

$$M = \frac{\Sigma n}{N}$$

where M is the mean, n are the individual observations, and N the total number of observations.

The three, mean, mode, and median, measure central tendency, and indicate the value around which the measures group. In the case of normal distribution they are the same in value and the replacement of the observed data by a fitted normal curve removes their difference. The differences in their value, in observed data, are used as a measure of the distortion or asymmetry of the distribution. The expression for skewness is

$$\text{Skewness} = \frac{3\ (\text{Mean} - \text{Median})}{SD}$$

where SD is the standard deviation.

Once the central tendency of the data is determined it is essential to know how the values lie with respect to this central tendency. There are several measures of variability which picture the spread of the observations.

The *Range* is the crudest form of measure for dispersion. It is the difference between the largest and smallest observation, and is subject to marked fluctuation. The range is indicated on Fig. 16.

The *Quartile* measures locate the middle half of the measures, but this must not be confused with the probable error which covers fifty per cent of the observations.

The *Mean Deviation* may be taken about either the mean or the median; for theoretical developments the median is used. The lower

part of Fig. 16 is a graph of the deviations of the data from the median. The average of these deviations, neglecting the sign, gives the mean deviation. This may be expressed by

$$MD = \frac{\Sigma d}{N}$$

where MD is the mean deviation, Σd is the sum of the individual deviations, and N is the total number of observations.

The *Standard Deviation* is the measure of deviation least affected by sampling, and gives a numerical value to the dispersion of the measures in the distribution. It also has an important mathematical significance and is an important statistical measure. It is the root mean square value of the deviations about the mean and is expressed by the equation

$$SD = \sqrt{\frac{\Sigma d^2}{N}}$$

where SD is the standard deviation, Σd^2 is the sum of the squares of the individual deviations, and N is the total number of observations.

The Coefficient of Variation is a value independent of the units on the scale, and is the ratio of the measure of absolute variability to the average from which the deviations were taken; it is expressed by

$$C = \frac{SD}{M} \times 100$$

where C is the coefficient of variation, SD the standard deviation, and M the mean.

Probable Error is another important factor in the variability measure. Fifty per cent of the measures will fall between plus and minus the probable error. But, as compared with the previous variability measures, this one is based on the assumption that the observed data are replaced by the distribution of the "universe" of which the observed data are a "random sample."

Since for the normal law there is a definite relationship between all the foregoing units, there is a fixed relation between the two important statistical units, which is

$$PE = 0.6745\ SD$$

where PE is the probable error and SD is the standard deviation. Therefore those measures marked PE and SD in Fig. 16 are for the

TABLE 18

FACTORS FOR PROBABLE ERROR

Size of Sample	Factor for Probable Error of Mean	Factor for Observation Rejection	Factor for Standard Deviation
4	0.4434	2.27	1.414
5	0.3720	2.44	1.291
6	0.3267	2.57	1.225
7	0.2935	2.67	1.183
8	0.2708	2.76	1.155
9	0.2523	2.84	1.134
10	0.2368	2.91	1.118
11	0.2233	2.96	1.106
12	0.2115	3.02	1.095
13	0.2077	3.07	1.087
14	0.1931	3.12	1.080
15	0.1864	3.16	1.074
16	0.1804	3.19	1.069
17	0.1750	3.22	1.065
18	0.1700	3.26	1.061
19	0.1655	3.29	1.057
20	0.1613	3.32	1.054
21	0.1573	3.35	1.051
22	0.1537	3.38	1.049
23	0.1503	3.41	1.047
24	0.1471	3.43	1.044
25	0.1440	3.45	1.043
26	0.1411	3.48	1.041
27	0.1384	3.50	1.039
28	0.1358	3.52	1.038
29	0.1333	3.54	1.036
30	0.1309	3.55	1.035

The coefficients in column two were computed from "Students" probability tables given in "Biometrika" Vol. 6, 1908, p. 1 and Vol. 11, 1917, p. 416. The coefficients in column three were calculated from a criterion of Chauvenet discussed by Meriman on page 106 in his "Method of Least Squares."

normal distribution having the same standard deviation as the observed data. The percentages opposite the lines show that portion of the observations that would fall in the classification shown.

3. *Reliability of Results.*—The reliability of the results is of considerable importance and is indicated by the probable error in the representative values for the data. These factors of reliability are all based on the presumption of normal error, and therefore on the normal curve.

For the normal curve there have been developed coefficients for reliability, but these assume a symmetrical distribution. "Student" has definitely shown that in the case of small samples there is a skew that precludes this assumption. Table 18 gives, in the second column, the factor by which the standard deviation must be multiplied to give the probable error of the mean, and the third column gives the factor

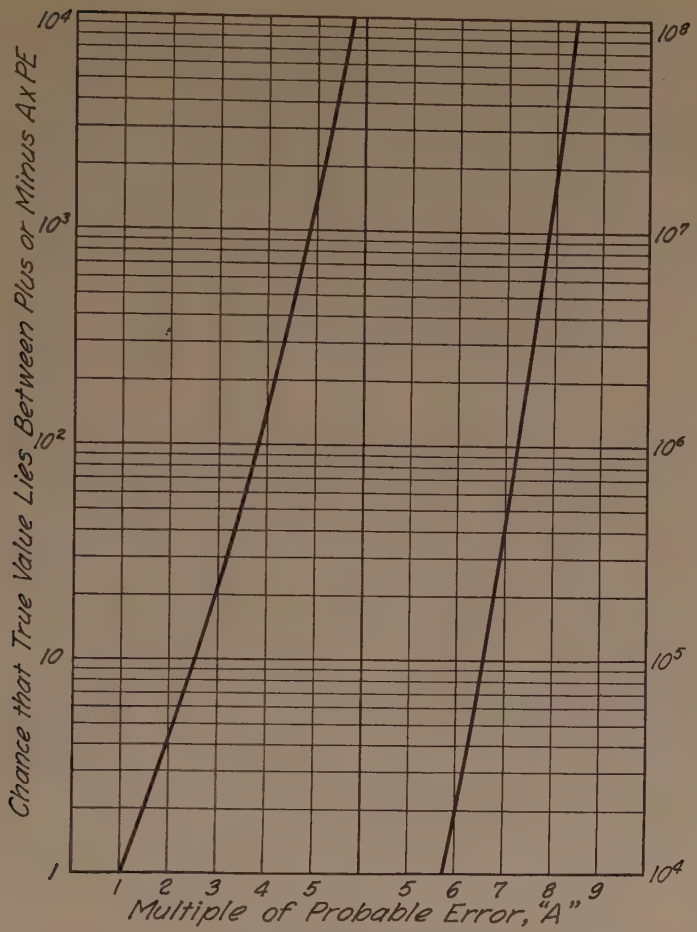


FIG. 17. CHANCE THAT TRUE VALUE LIES BETWEEN $\pm$ *A* PROBABLE ERROR

that the ratio of the largest deviation to the probable error must not exceed.

The probability is calculated from

$$P = \frac{2N - 1}{2N}$$

where N is the size of the sample, and the factor is determined from any probability table where the percentage of area under the probability curve is expressed in units as a ratio of the deviation to the probable error.

TABLE 19

EXAMPLE OF STATISTICAL SOLUTION

Number of Observations N	Dielectric Strength kv.	Deviation from Mean d	Square of Deviation from Mean d^2
1	61.2	−13.94	194.3236
2	61.5	−13.64	186.0496
3	62.5	−12.64	159.7696
4	66.5	− 8.64	74.6496
5	68.7	− 6.44	41.4736
6	69.0	− 6.14	37.6996
7	69.1	− 6.04	36.4816
8	70.6	− 4.54	20.6116
9	73.7	− 1.44	2.0736
10	74.7	− 0.44	0.1936
11	77.0	+ 1.86	3.4596
12	77.5	+ 2.36	5.5696
13	77.5	+ 2.36	5.5696
14	79.4	+ 4.26	18.1476
15	79.7	+ 4.56	20.7936
16	82.9	+ 7.76	60.2176
17	83.0	+ 7.86	61.7796
18	84.5	+ 9 36	87.6096
19	88.4	+13.26	175.8276
20	95.4	+20.26	410.4676
	1502.80	147.80	1602.77

Median=the average of 10 and 11 ..... 75.85 kv. (76)
Mean=column 2 divided by 20 ..... 75.14 kv. (75)
Mode=75.14—3(75.14—75.85) ..... 77.27 kv. (77)
Range=95.4—61.2 ..... 34.2 kv. (34)
Mean Deviation, MD=column 3 divided by 20 ..... 7.40 kv.
Standard Deviation, SD=square root of column 4 divided by 20 ..... 8.95 kv.
Corrected by Factor Table 17 ..... 9.44 kv.
Probable Error, PE=0.6745×9.44 ..... 6.37 kv.
50 per cent of the observations will lie between 68.8 kv. and 81.5 kv. (compare with results by graphical method Figure 16)
3 SD=3×9.44 ..... 28.32 kv.
99.73 per cent of the observations will lie between 46.8 kv. and 103.5 kv.
C=9.44/75.14 ..... 0.13
Skewness ..... −0.24
Permissible Deviation Factor for N = 20, Table 17, is 3.32
Maximum Deviation in observations=20.26 kv.
Probable Error of the observations=6.37 kv.
20.26 divided by 6.37 = 3.18 kv.
therefore all observations are satisfactory.
Probable Error of the Mean, PE_M, from Table 17 =0.1613×8.95 =1.44
50 per cent of the averages for samples of 20 will fall between 73.7 kv. and 76.6 kv., which is an accuracy as great at that of the testing.

In order that there may be a satisfactory degree of reliability conservative practice demands that the coefficient under consideration shall be four or five times the probable error of that coefficient. This means that the chance of the true value lying between plus and minus the computed probable error is 1 to 142 or 1 to 1310. Figure 17

gives a graph for the determining of chance when the multiple of the probable error has been determined.

The fourth column gives the factor by which the standard deviation should be multiplied for determining the standard deviation for a small sample. This correction should not be used at the same time as the correction factor in column two. These factors have been calculated after the work of Pearson, "Biometrika" Vol. X, 1915, page 522, using the following expression:

$$SD = \sqrt{\frac{N}{N-2}}\, SD_0$$

where N is the size of the sample, SD is the true standard deviation, and SD_0 is the standard deviation obtained from the observed data. This value of SD should be used in finding the limits within which 50, 68.27, 95.54 and 99.73 per cent of the observations fall.

4. *Examples.*—In order that the statistical terms may be clearly set forth, Table 19 presents a complete problem, using the case shown in Figs. 14 and 15.

It is not desirable to place emphasis on the decimal, as the whole numbers are sufficiently accurate for the samples used and the possible accuracy of testing.

Table 20 gives data for the determination of rank correlation for a sample which was investigated both for strength in tension and dielectric strength. The number 1 indicates superior strength above all others, 2 second in rank, and so on up to sixth.

This information is then arranged in tabular form and the difference in rank determined by subtracting one rank from the other, as follows:

Body	Rank for Strength in Tension	Rank for Dielectric Strength	Difference	
			Pos.	Neg.
A	3	3	0	0
B	1	1	0	0
C	2	4	0	2
D	4	2	2	0
E	5	5	0	0
F	6	6	0	0
			Sum 2	

TABLE 20

DATA FOR DETERMINATION OF RANK CORRELATION

Body	Rank for Strength in Tension	Rank for Dielectric Strength
A	3	3
B	1	1
C	2	4
D	4	2
E	5	5
F	6	6

Only the sum of the positive differences will be used in the determination of the correlation coefficient. The following is the expression for determination of the coefficient:

$$r = 2 \cos\left[\frac{\pi}{3}\left(\frac{6 \text{ Pos. diff.}}{N^2 - 1}\right)\right] - 1$$

$$r = 2 \cos\left[(1.0472)\left(\frac{6 \times 2}{36 - 1}\right)\right] - 1$$

$$r = 0.87$$

This indicates a direct relationship, and is probably very satisfactory. When the correlation is 1 the relationship is perfect, while zero denotes no relationship whatsoever. The rank correlation is more satisfactory for small samples than the moment method, but it should not be used where the sample exceeds thirty specimens.

Appendix B

Bibliography

The following bibliographies contain general information on the subject of dielectrics. According to the order in which they are listed will be found the literature covering the general field of dielectrics from the earliest investigation to the present day.

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No.	Year	Author	Title
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2	1910	Weimer, G. O.	The Effect of Temperature on the Electric Strength of Porcelain Elec. Rev. and West. Elec'n, Vol. 57, p. 1179
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No.	Year	Author	Title
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